UNITED STATES DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
WASHINGTON, D. C.
H. H. BENNETT, CHIEF

CHANNEL WAVES SUBJECT CHIEFLY TO MOMENTUM CONTROL

Robert E. Horton

Contribution from

Division of Research, Soil Conservation Service
and
Horton Hydrologic Laboratory
Voorheesville, N.Y.

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E R R A T A

CHANNEL WAVES SUBJECT CHIEFLY TO MOMENTUM CONTROL
By Robert E. Horton, May 1938.

Contents, 11th line - change Carstanjon to Carstanjen.
14th line - insert another "h" in Forchheimer.

Page 12, first line should read: h_v = velocity head = $\frac{v^2}{2g}$.

Page 13, 4th line should read: w = weight of water per cubic foot.

Page 20, in equation 16, the third term should be preceded by an equation sign (=) instead of a plus sign (+).

In equation 19, the second term under the radical sign should be squared.

In equation 21, the second term under the radical sign should be contained within parenthesis.

Page 26, first line of last paragraph, Forchheimer misspelled twice; another "h" should be inserted.

In equation 30, change "w" to "W".

Page 27, 5th sentence, the word "agin" should be "again".

In equation 31, change "w" to "W".

Forchheimer misspelled 4 times on this page; another "h" should be inserted.

Page 28, Forchheimer misspelled twice; another "h" should be inserted.

Page 29, column 8 should read $\sqrt{gd_1}$.

Page 31, Forchheimer misspelled in last column of tabulation; another "h" should be inserted.

Page 33, Forchheimer misspelled 3 times; another "h" should be inserted.

In paragraph numbered 5, third sentence, typographical error, the word "velow" should be "below".

Page 35, second paragraph, 4th line, the radical sign is missing for gd_2; this should read $\sqrt{gd_2}$.

Page 38, first line, change "w" to "W".
Equation 33 - same.
Equation 34 - same.

Page 40, 8th line, change "w" to "W".

Page 47, Forchheimer misspelled 5 times; another "h" should be inserted.

Page 48, Forchheimer misspelled 3 times; another "h" should be inserted.

Page 50, paragraph (b) under paragraph numbered 10, typographical error - transposition in spelling of word "channels".

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CHANNEL WAVES SUBJECT CHIEFLY TO MOMENTUM CONTROL

Robert E. Horton

INTRODUCTION - Problems involving the motion of increment and decrement waves in more or less uniform channels have arisen at various times in the author's professional practice, notably in connection with the design of the Tonawanda-Lockport level of the New York State Barge Canal and in connection with the proposed regulation of diversion from Lake Michigan through the Chicago Drainage Canal. In order to provide data for a better solution of such problems than was otherwise available, the series of experiments described below was carried out, beginning in 1924, at the author's laboratory. Grateful acknowledgment is made to the Sanitary District of Chicago for authorizing the continuation of the experiments in 1925 and 1926 and defraying a large portion of the cost.

The work was carried out by the author and members of his staff intermittently as gaps in professional practice permitted. Acknowledgment is made to H. R. Leach, M.A.S.C.E. and Richard Van Vliet, M.A.S.C.E., and to George E. Cook and Edward M. Dooley as observers, and James Erwin as mechanic. Van Vliet, Cook, and Dooley also participated in the preparation of this paper. In particular, credit is due Van Vliet for work in computing comparative velocities by various formulas and in preparation of tables and diagrams.

Instead of attempting to apply the resultant experiments empirically or otherwise to the derivation of a wave velocity formula, they

have been compared with the results given by existing formulas, particularly those of Bazin and Darcy, Leach and King, and Koch and Carstanjen.

Because of the complexity of the subject and the difficulty of making statements of results which would be accurately intelligible without reading the paper, conclusions are reserved to be given at the end of the discussion. It may be mentioned, however, that the experiments indicate a gradual transition from waves subject to momentum control to those subject chiefly to friction control as the wave length increases, and a much more comprehensive series of experiments is needed than has ever been carried out to fully elucidate the subject.

It is pointed out that the degree to which channel waves are subject to friction control is apparently related to two independent variables, the wave length or, perhaps better, the ratio of wave length to wave height, and the ratio of the hydraulic radius of the channel to the wave height.

It is hoped that this paper may point the way to a more adequate study of the whole subject of velocity of channel waves, a study which will avoid gaps between the different related cases and will provide connective tissue to correlate all types of channel waves with the independent variables by which they are controlled.

APPARATUS - The experiments were conducted at the author's laboratory on Vly Creek, near Voorheesville, N. Y. A pond, about 3 acres

in area, formed by the laboratory dam, afforded an abundant supply of water at nearly a constant level. Water was conducted from the pond to the experimental apparatus by means of a short sluice with an adjustable gate. The apparatus comprised three essential parts: the orifice box, the channel or flume, and the recording gages. A general plan of the apparatus is shown on figure 1.

ORIFICE BOX - The orifice box, constructed of pine, 4 feet in height and 2 feet square inside, is shown on figure 2. A spillway was provided on the side opposite the orifice. Just sufficient water was admitted through the sluice during an experiment to maintain a slight overflow, so that the head on the center of the orifice was sensibly constant. A brass orifice plate, 1/8 inch in thickness, contained a square-edged opening 0.1 foot wide and 8-1/4 inches in length. A sliding metal shutter was provided (see detail) by means of which the length of the orifice opening could be cut down to any desired degree. The shutter plate could be moved either by a lever or by a screw. Lever operation was used in cases where an instantaneous increase or decrease of flow was desired. The operating lever slid along a stop-arm into which a metal pin could be fitted tightly, through holes uniformly spaced, thus providing a series of fixed openings which could be exactly duplicated. The lever was set firmly against the pin in the position desired for initial flow. The pin was then moved to the position required to make the desired increment or decrement of flow, and a quick movement of the lever to the new position thus gave an instantaneous predetermined increase or decrease of discharge.

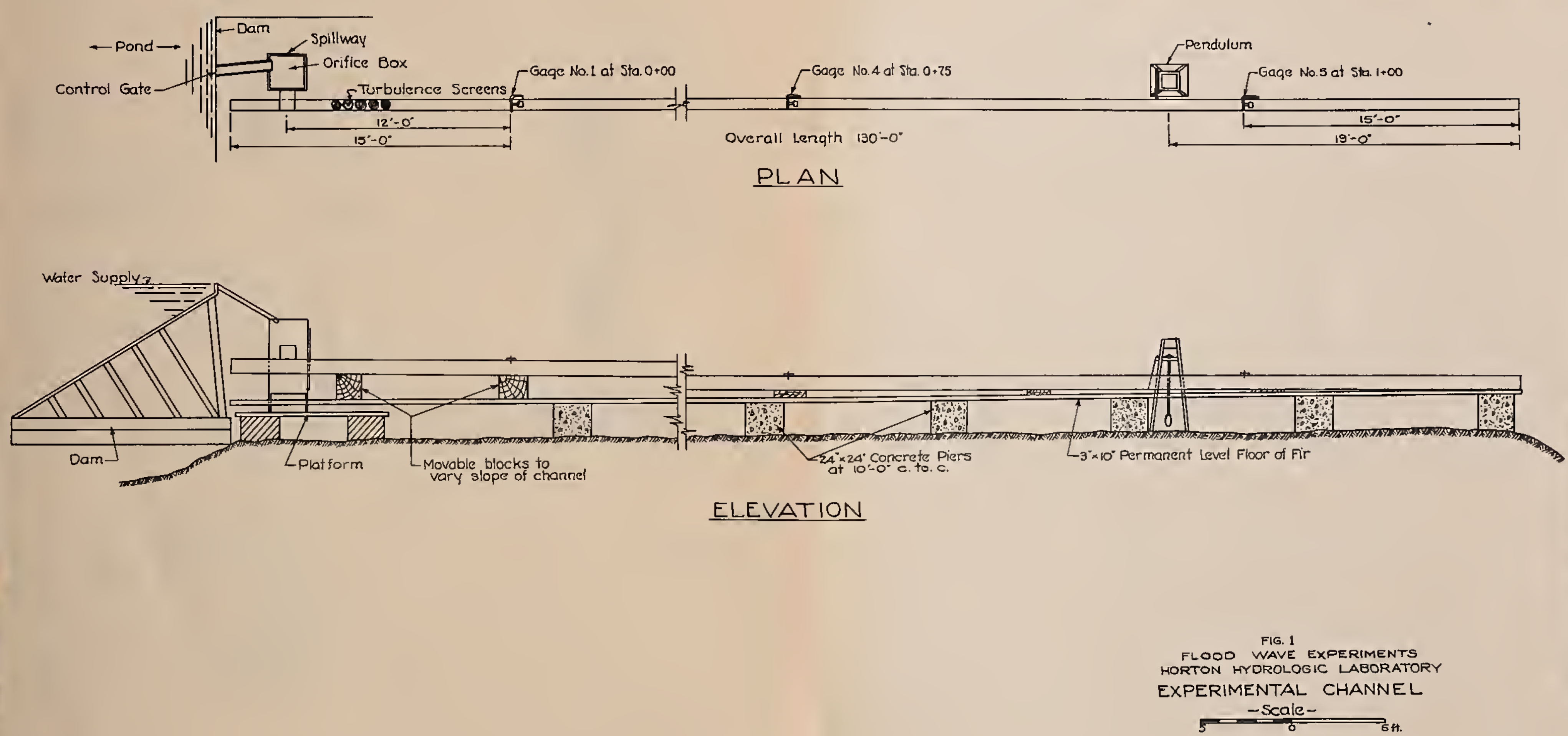

FIG. 1
FLOOD WAVE EXPERIMENTS
HORTON HYDROLOGIC LABORATORY
EXPERIMENTAL CHANNEL

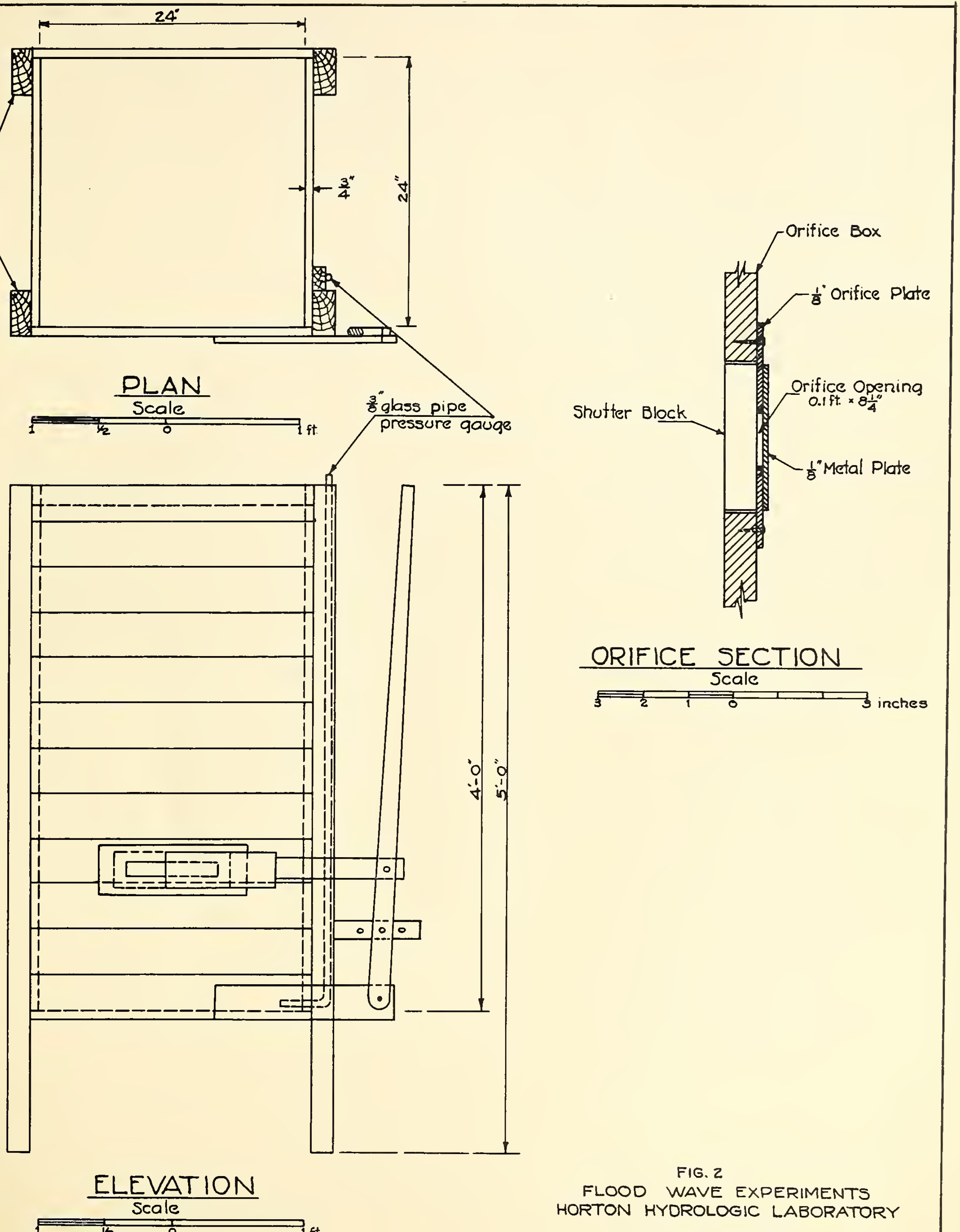

FIG. 2
FLOOD WAVE EXPERIMENTS
HORTON HYDROLOGIC LABORATORY

ORIFICE BOX

To obtain a gradual increase or decrease of discharge, the lever was disconnected and a screw movement (not shown on drawing) was used to operate the shutter plate. By giving the screw a predetermined number of turns in a given time, the rate of flow could be changed the desired amount at any desired rate.

Before the experiments were performed, the orifice was accurately calibrated for various heads and openings by closing the lower end of the experimental channel and using the channel as a tank to measure the outflow from the orifice for a given time. It was found that the discharge coefficient of the orifice was not constant but decreased slowly as the opening decreased, as illustrated by the following experiments made September 13, 1924:

Length of opening - feet:	0.105	0.200	0.305	0.406
Mean head - feet:	2.224	2.240	2.215	2.200
Coefficient:	0.619	0.610	0.6075	0.590

A slight change was made in the orifice in 1926. This affected the coefficient and a new calibration curve was obtained and used in experiments subsequent to that date.

EXPERIMENT CHANNEL - The channel or experimental flume, 130 feet in length overall, rested on a level table consisting of 3 x 10-inch Oregon fir plank, supported by 24-inch square concrete piers, spaced at 10-foot centers (fig. 1.)

The channel (see section) had a 1-1/4 X 5-5/8-inch clear white pine bottom plank, with 7/8 X 8-7/8-inch (inside measurement)

vertical sides. When in use, either level or at a given grade, the grade was accurately fixed by means of a Y-level, and suitable grades were obtained by placing movable blocks on the table underneath the channel, the thicknesses of the blocks decreasing proceeding downstream. The side and bottom joints of the channel were staggered and great care was taken to obtain smooth joints. Even then it was found that slight roughnesses, particularly at the joints, tended to throw off reflected waves. These were usually small in height relative to the traveling wave and did not greatly interfere with obtaining accurate profiles of the latter.

The channel bottom as originally constructed was practically 5-5/8 inches or 0.473 foot in width. There were, however, slight variations in width, as shown by micrometer width measurements. Also, the width varied slightly according to the condition of the channel when an experiment was being performed. The bottom width was sensibly constant. The side boards tended to warp outward when dry, increasing the width slightly, proceeding up from the bottom. Micrometer measurements of width at bottom and at 5 inches above bottom were made at various times. The results of such a series of measurements, at 5-foot longitudinal intervals, made November 10, 1926, is shown by the following figures.

Table 1. - Channel widths

Station	Number of points	Average width - inches	
		Bottom	5 inches depth
(1)	(2)	(3)	(4)
0 - 10	3	5.63	5.80
15 - 35	5	5.65	5.68
40 - 60	5	5.66	5.73
65 - 85	5	5.66	5.72
90 - 100	3	5.70	5.75

While there was a tendency for the width to slightly exceed 5-5/8 inches, the difference was within the limit of the errors of observations, and to simplify the computations and avoid the necessity, which would otherwise have arisen, of making a separate channel flow calibration for each experiment, the experimental data have all been computed on the basis of a channel width of 5-5/8 inches.

Variation in channel width does not affect wave velocities, which were directly measured. It does affect slightly the velocities v_1 and v_2, which were used in calculating wave velocities for comparison with those observed. The error in the calculated velocities is negligible but tends to make the calculated velocities slightly too small.

GAGES - Five gages were placed along the channel, spaced 25 feet apart. The orifice discharged into the channel 12 feet upstream

from Gage 1, and in this 12-foot length one or more rolls of coarse mesh wire screen were placed. These were intended to produce approximately a normal velocity distribution in the channel section, with a minimum impedence of flow. The 15 feet of channel downstream from the lower gage (Gage 5) was unused excepting to eliminate the drop-down curve in open-end experiments. For experiments on a level channel, barriers of different heights were placed in the downstream end.

In order to obtain precisely simultaneous records of water levels in the channel at all five gages, a 7/8 x 1-1/4-inch pine draw-bar was run along outside the channel on one side and flush with the bottom. This was permitted to slide easily over supports attached to the bottom of the channel. At each gage a vertical "Beaverboard" gage-board was mounted on this draw-bar and held vertically against the outside wall of the channel by loose clips, so that it could slide freely. Suitable record sheets were mounted at the gage-boards with thumbtacks (fig. 3).

Between Gages 4 and 5 a heavy lead pendulum with knife edge supports was contained in a sturdy wooden frame. The pendulum made one complete cycle in about 2.3 seconds and was actuated by hand during the experiments. The pawl at the top of the pendulum-bar engaged the teeth of a ratchet wheel, so designed as to pull the draw-bar forward a fixed amount at uniform intervals of about 2.3 seconds, by means of a chain wound around the shaft of the ratchet

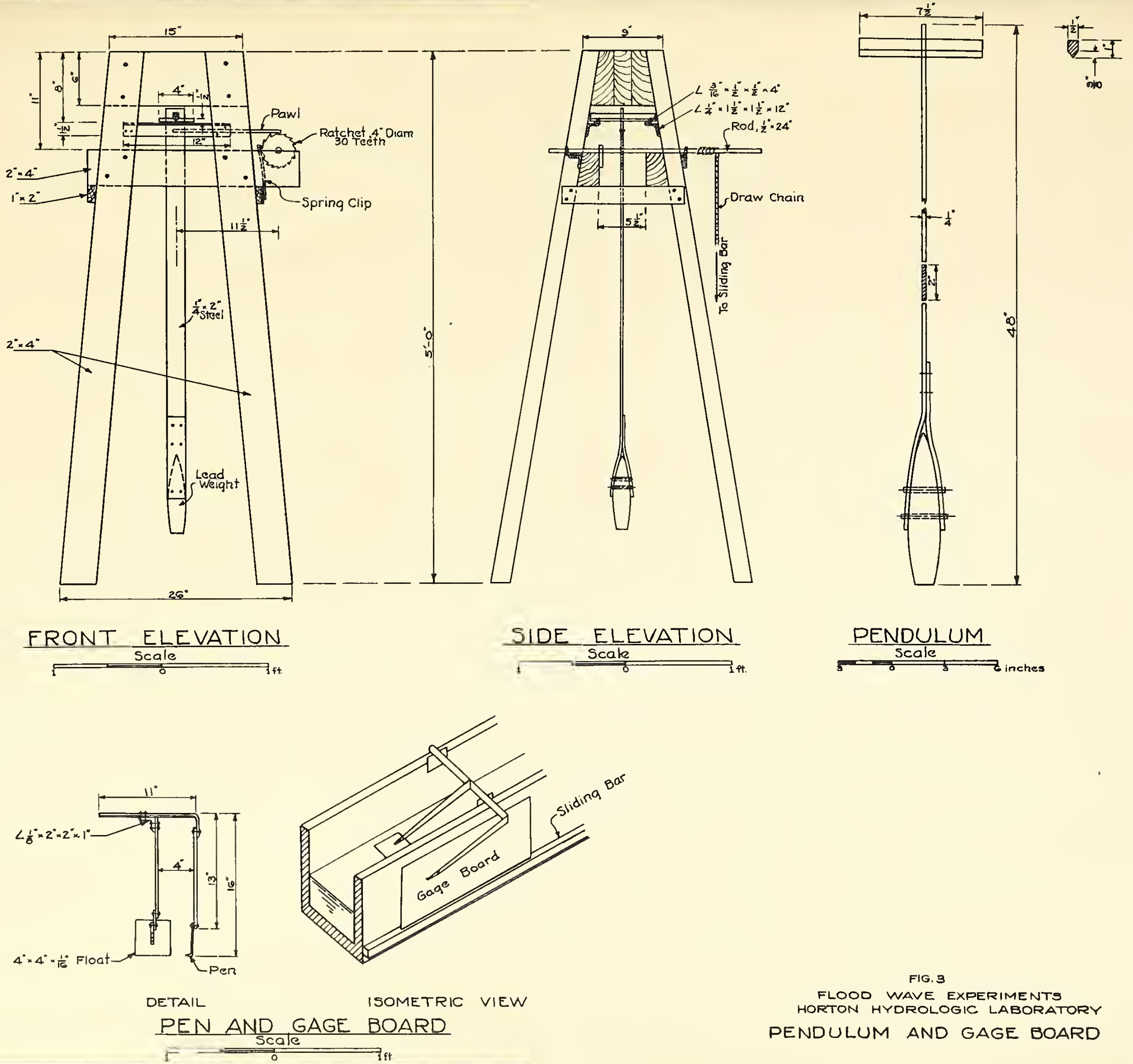

FIG. 3
FLOOD WAVE EXPERIMENTS
HORTON HYDROLOGIC LABORATORY
PENDULUM AND GAGE BOARD

wheel. The fluctuations of water level in the channel were transferred to the record sheet at each gage by a cross-bar having two arms (fig. 3). The ends of the metal cross-bar rested on knife edge supports on the sidewalls of the channel. One arm, mid-width of the channel, had a float or "skip" on its end, intended to rise and fall simultaneously with the water. At the outset a light balsa wood float was used. In spite of its lightness, its inertia prevented it from following closely the abrupt changes of water level, and a thin curved metal skip was substituted (fig.3). This at all times floated on the water surface, with the lower edge slightly submerged, and a correction was made for submergence by means of careful, direct measurements from bench marks established at each gage. The motion of the skip was transferred through the cross-bar to the outer arm, which bore on its end a pen of the type used in "Friez" recording instruments. The radius of the outer arm, from knife edge to pen point, was 16 inches. Thus as the water rose and fell in the channel, the pen marked an arc of a circle of 16 inches radius on the record sheet. Meanwhile the record sheet moved forward by abrupt increments, at uniform time intervals. In this way a stepped hydrograph of the flood wave was obtained at each gag simultaneously. By this method the original graph for a given gage shows cross-marks corresponding to the fluctuation of water during the interval between successive advance of the draw-bar and the times are identical on all the five gage graphs for a given experiment. The position of the pen on the sheet was noted at frequent intervals at each gage at recorded times,

and from these data and the time marks on the sheets, the graph could be reproduced with reference to simultaneous times for the different gages with great accuracy. When an experiment was completed, another was started without stopping the pendulum. By changing the position of the pen on the record sheet, graphs of six experiments could be obtained for a given gage on a single sheet. Figure 4 is a sample reproduced from an original gage sheet.

The graphs for the different gages, for each experiment, were later replotted as smooth curves on rectangular cross-section paper.

CALIBRATION OF CHANNEL - For the purpose of analysis and comparison of the results, it was necessary to know the stage-discharge relations for the channel with different rates of inflow from the orifice box. For each series of experiments the channel was calibrated by running a series of different quantities of uniform flow through it, as determined by the opening of the orifice, and measuring the corresponding water levels at the five gages. A calibration by this method was made for each slope of the channel and, in case of a level channel, for each series of experiments with a different height of end weir to control the drop-down curve at the downstream end of the channel. In addition, before each experiment, a constant volume of water was allowed to flow through the channel for a time to establish uniform flow corresponding to the initial depth used in the experiment. Gage readings taken in this connection, together with the corresponding discharges, furnish an additional set of data for calibration of the channel with different volumes of flow.

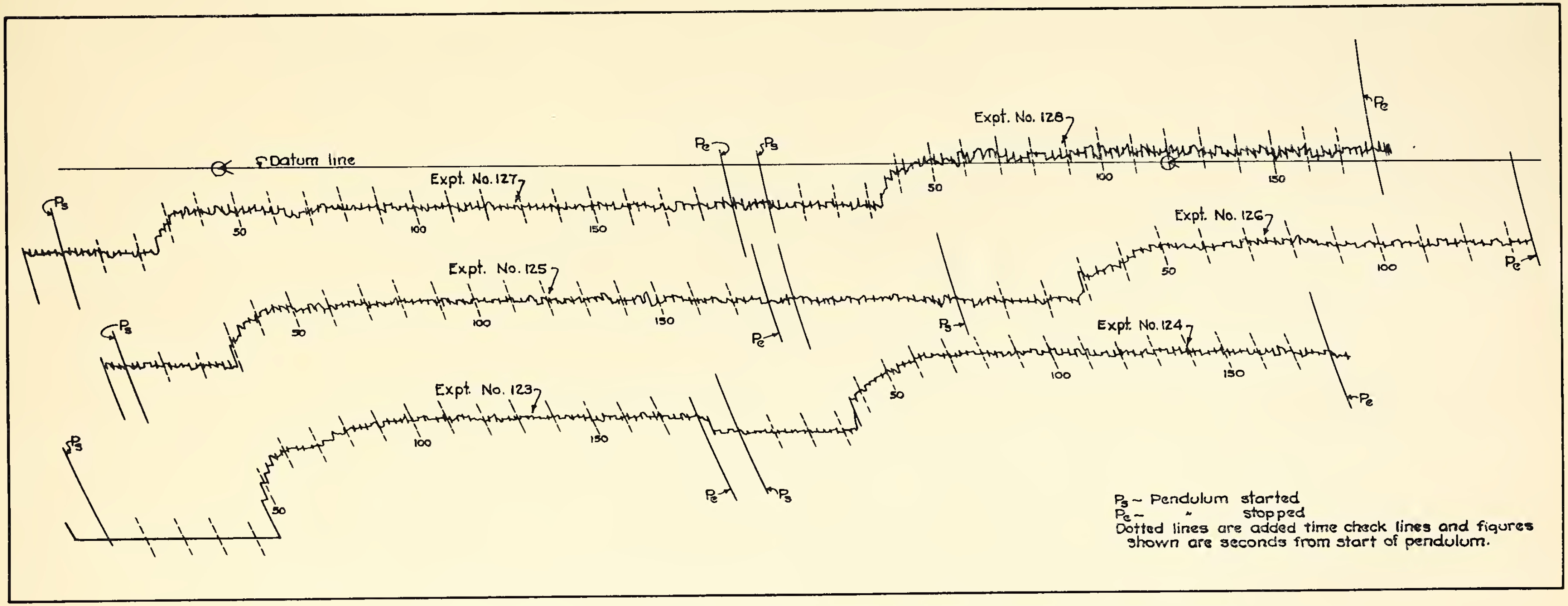

FIG. 4 -SAMPLE OF ORIGINAL GAGE GRAPH. GAGE 3, SERIES C, NOV. 12, 1925

Figs. 5 and 6 show the stage-discharge relations of the channel for the different slopes and conditions corresponding to different series of experiments. From the stage-discharge curves, stage-velocity curves were obtained by dividing the discharge for a given stage by the cross-section area of the channel. In these computations a constant channel width of 5-5/8 inches was assumed. Figures 7 and 8 show the resulting stage-velocity relation curves. These give the mean velocity in the channel corresponding to the average of the stages at gages 1 to 5, inclusive. The discharges and velocities corresponding to stable flow with different depths in the channel, as given in the tables of results of the experiments, were derived from these two series of diagrams. Values of the coefficient of roughness n were obtained from these discharge curves for various stages, as shown by table 2. The values of n range from about 0.010 to 0.014 and decrease as the depth of flow increases, indicating greater roughness of the bottom than sides of the channel. The values of n also indicate that the flow was turbulent even with depths less than 0.01 foot. Where a level channel was used, with a weir in the lower end, this did not wholly correct (or else overcorrected) for the effect of the drop-down curve, and values of n were not determined.

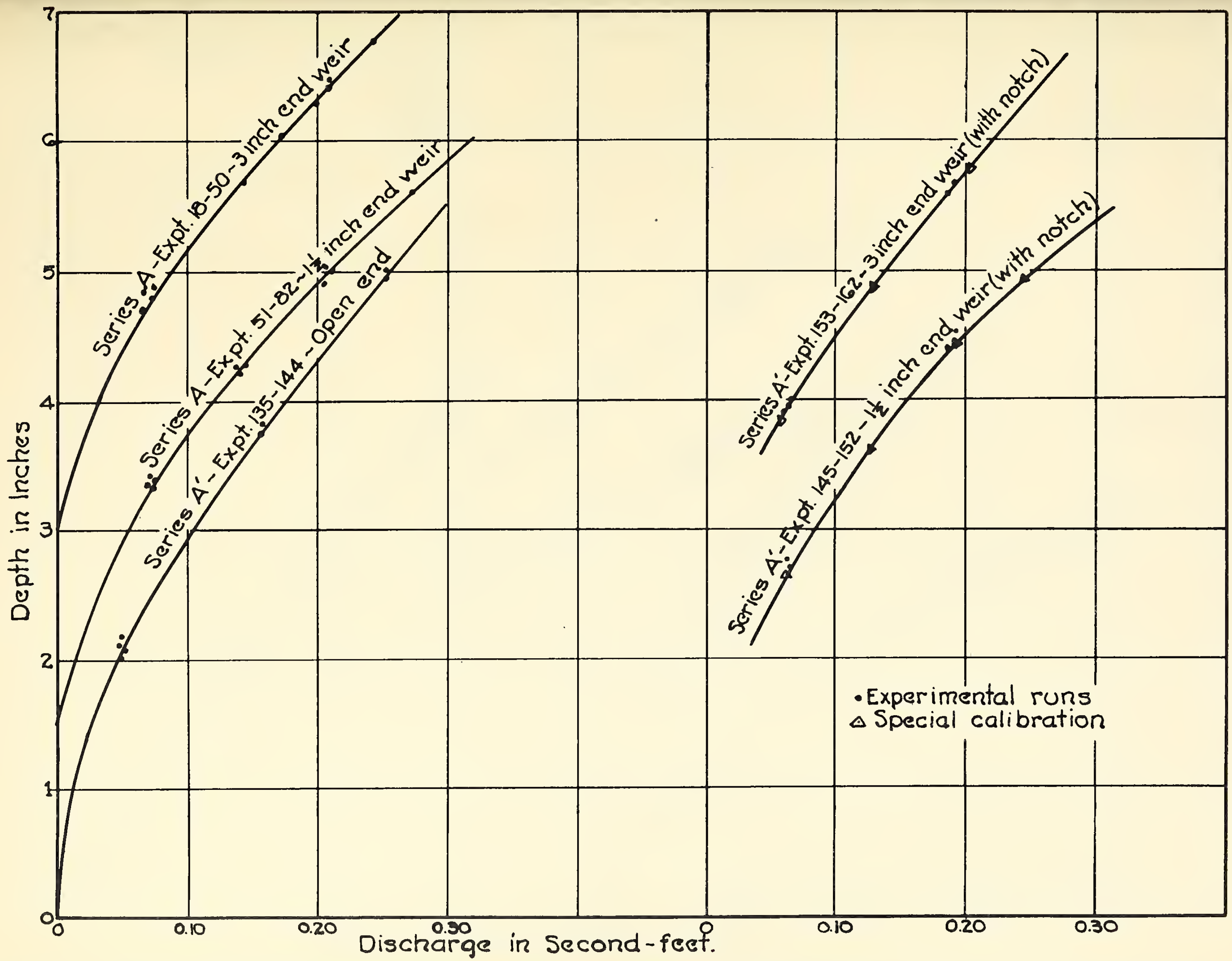

FIG. 5 ~RELATION BETWEEN DEPTH AND DISCHARGE FOR EXPERIMENTS WITH LEVEL FLUME

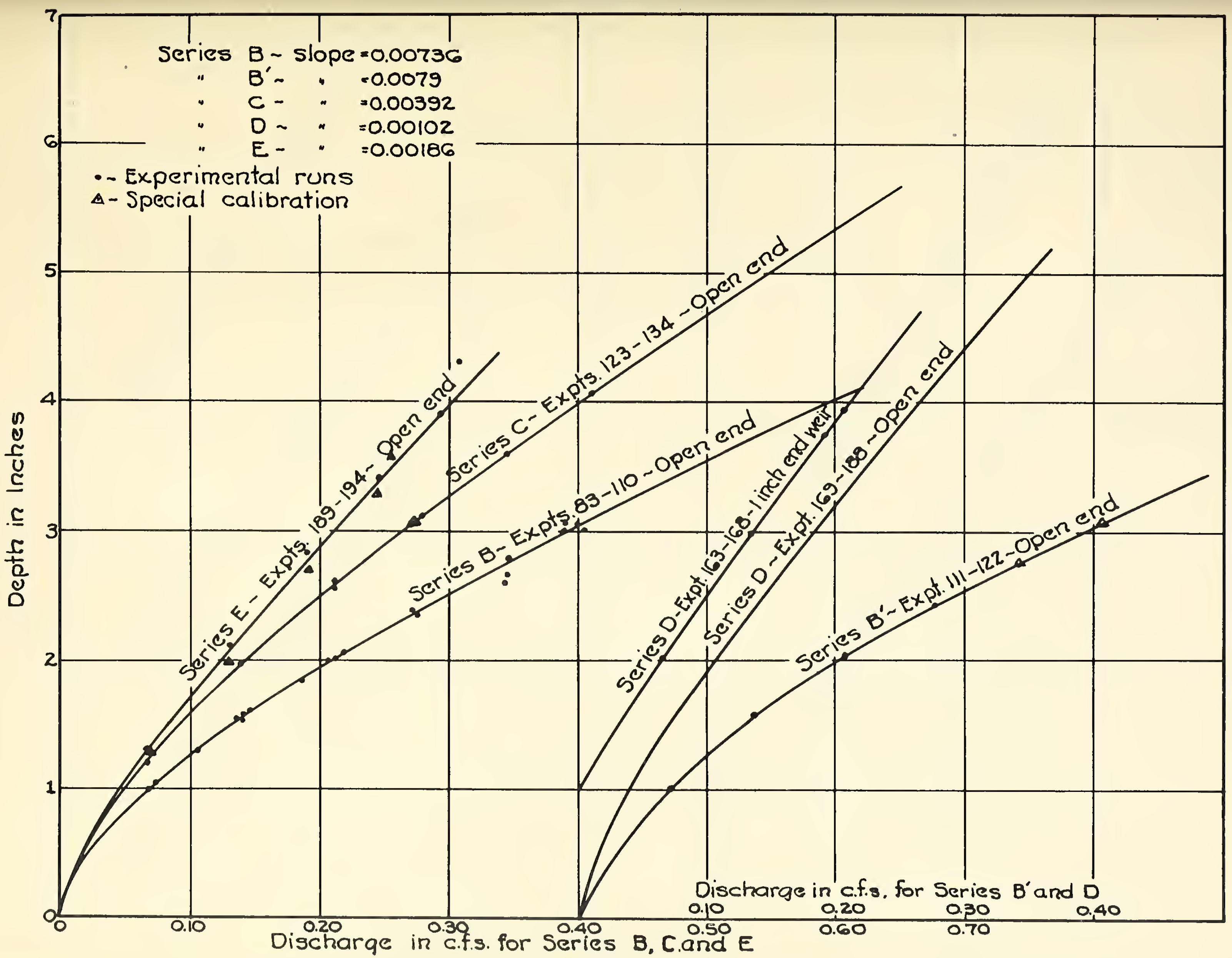

FIG. 6 - RELATION BETWEEN DEPTH AND DISCHARGE FOR EXPERIMENTS WITH SLOPING FLUME

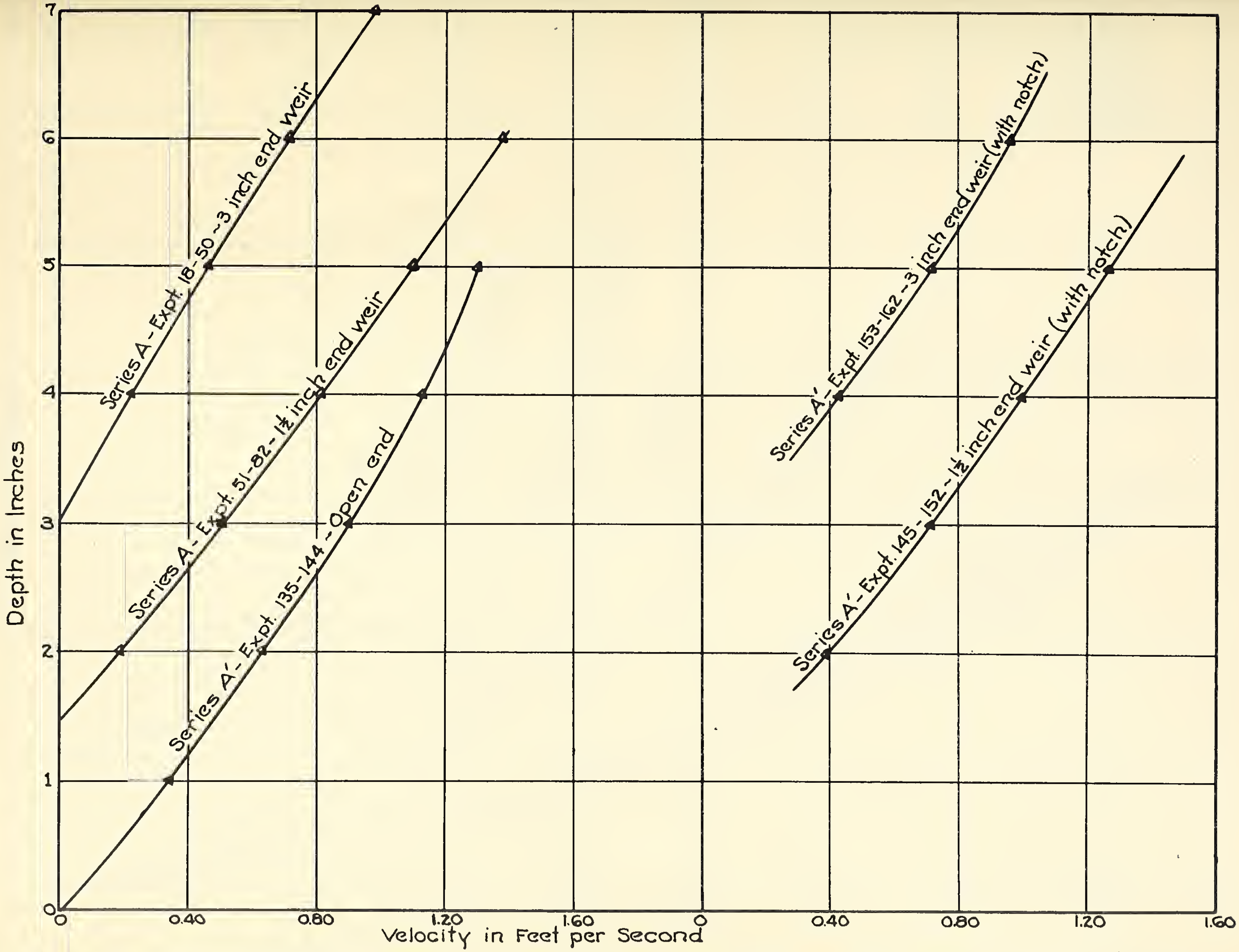

FIG. 7 - RELATION BETWEEN DEPTH AND VELOCITY FOR EXPERIMENTS WITH LEVEL FLUME

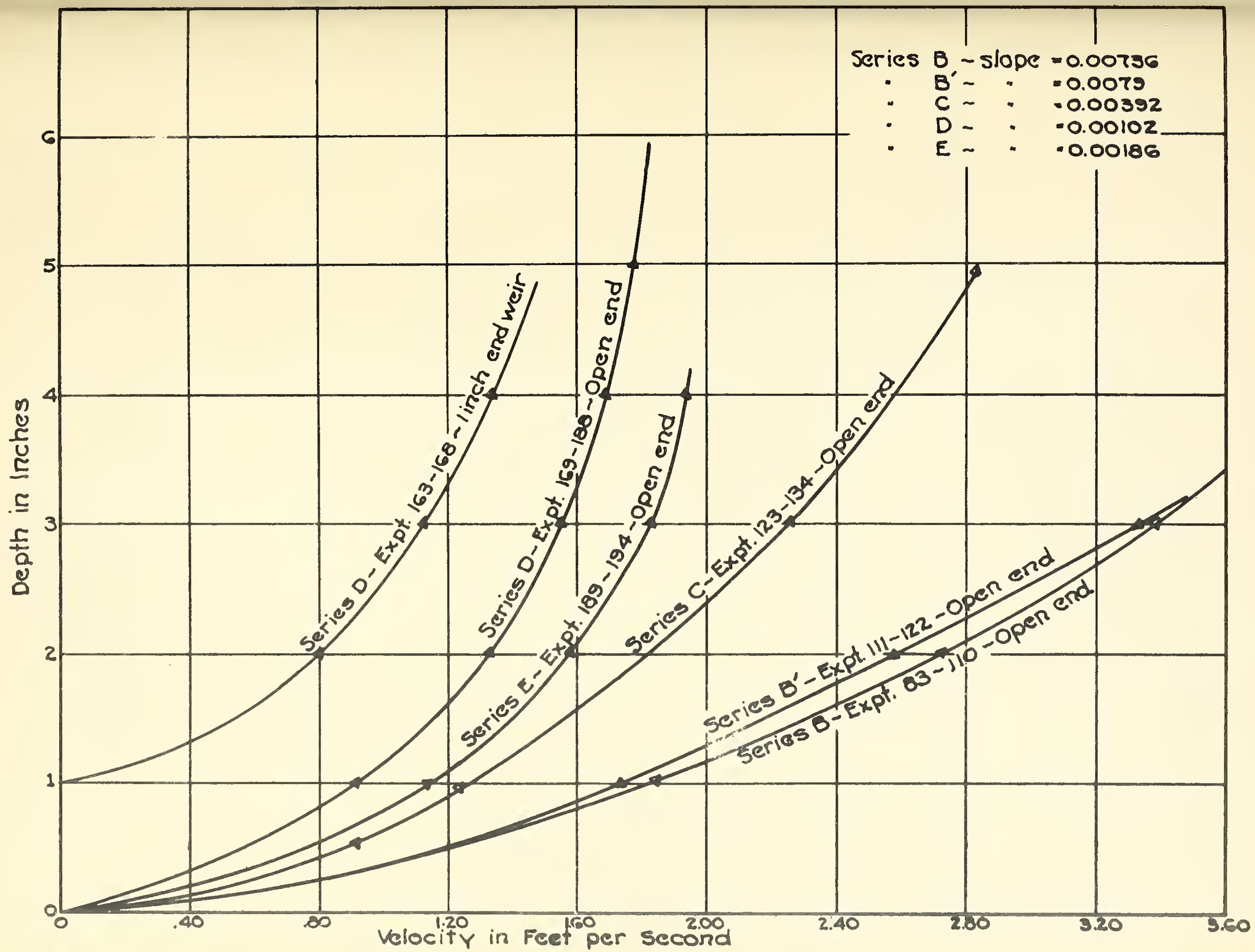

FIG. 8 ~ RELATION BETWEEN DEPTH AND VELOCITY FOR EXPERIMENTS WITH SLOPING FLUME

Table 2 - Values of n in Manning's formula, $n = \frac{1.486}{v} \sqrt{s} \; d^{2/3}$,

for experiments without end weir.

Series	s	$\sqrt{s}$	v = 0.40 f.p.s.		v = 0.80 f.p.s.		v = 1.20 f.p.s.	
			d, feet	n	d, feet	n	d, feet	n
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
B	.00736	.08579	.00833	.013	.01999	.0078	.04166	.0128
B'	.0079	.08888	.00916	.0143	.02249	.0132	.04332	.0138
C	.00392	.06265	.01166	.0119	.03540	.0125	.07497	.0140
D	.00102	.03182	.02666	.0106	.06831	.0101	.13411	.0105
E	.00186	.04317	.01833	.0115	.04665	.0106	.08996	.0108

NOTATION - In the discussion of the results of these experiments the following notation is used. Quantities are in foot-second units excepting that in the tables of experiments the depths in the channel are given as they were measured - in inches. Directions and velocities are positive in the direction of the initial flow.

d_1, v_1 = initial depth and velocity.

d_2, v_2 = depth and velocity after the wave has passed.

d_c = depth at crest of wave.

d_h = depth at heel of wave.

d_k = depth at cope of wave.

d_t, v_t = depth and velocity at bottom of a trough wave.

Δ_d, Δ_q = increments of stage and discharge.

h = stream stage with steady regimen.

h_v = velocity head = $\frac{v^2}{2g}$

H = head or depth on a weir or orifice.

l_b = length of back face of a wave.

l_c = length of crest portion of a wave.

l_f = length of face or front of a wave.

l_w = total length of wave.

L, L_1, L_2, etc. = lengths of stream reaches, etc.

m = exponent in stage-discharge relation formula.

M = momentum.

n = Manning's coefficient of roughness.

P = wetted perimenter of a cross-section.

q = discharge past a given section per unit width of channel.

Q = discharge past a given section.

R_0 , R_1, R_2, etc. = hydraulic radii of cross-section.

S_c = slope of stream channel.

S_f, S_b = slope of front and of back faces of a wave.

u = velocity of travel of the front or face of a wave.

u' = velocity of travel of rear face of a two-faced wave.

u_c = velocity of crest of wave.

u_g = maximum velocity of wave.

u_h = velocity of heel of wave.

u_t = velocity of toe of wave.

v = mean velocity, by Manning formula, for neutral flow at any given depth and slope.

V = total volume of a wave.

V_c = Belanger's critical velocity.

W, W_0, W_1, W_2, etc. = width of water surface.

W = weight of water per cubic foot

x = distance measured horizontally.

EQUATIONS OF COMPARISON - The results of the author's experiments have been compared with the equations of Bazin and Darcy, Leach and King, and Koch and Carstanjen, as given on table 5.

Bazin and Darcy - Bazin and Darcy[1] did not attempt to derive an equation from their experiments but assumed that for waves traveling in the same direction as the initial current, as in the author's experiments, the wave velocity would be nearly the sum of the initial velocity v_1 and the velocity for a wave in still water, $\sqrt{gd_2}$. They found empirically that the computed velocity in general agreed well with the observed velocities in their experiments if, instead of adding or subtracting v_1 in case of waves traveling downstream and upstream, respectively, they added the fraction $3/5v_1$ for waves traveling downstream, and subtracted $2/5v_1$ for waves traveling upstream. In many of their experiments the wave broke or formed a whitecap, and there is some question whether Bazin and Darcy did not intend the correction factor, as last described, to be applied only in cases of waves with breaking crests. In the tables giving the results of their own experiments they invariably use the full value

---
1 Bazin (and Darcy). Experimental researches on the propagation of waves (Fr.). Mem. Sav. Ac. Sc. Inst. Imper. de France, XIX, pp. 495-652; Paris, 1865.

of v_1 in calculating the wave velocity.

It is not entirely clear whether Bazin and Darcy intended that the velocity which they designate u, which is to be added to $\sqrt{gd_2}$ to obtain the wave velocity, should be the initial velocity of the current or the velocity of the current after the wave has passed, i.e., whether it should be v_1 or v_2. They describe it merely as the "mean velocity of the current." The tables of their results give neither the value of v_1 nor of v_2 and it is difficult to determine precisely how their calculated velocities were obtained.

There are reasons for assuming that it was the initial velocity of the current before the wave passed which they intended should be used. One reason is that a comparison of the wave velocities calculated on this basis with the observed velocities shows much better agreement than if the calculated velocities are obtained by the use of v_2.

In the author's experiments the wave did not break or form whitecaps. In order to determine which of the two equations of Bazin and Darcy, $u = \sqrt{gd_2} + \frac{3}{5}v_1$ or $u = \sqrt{gd_2} + v_1$, better applies to the author's experiments, a comparison was made between the averages of the results for each group of experiments and the velocities determined by each of these two formulas, as shown by table 3. Column (4) of this table shows that for all but triangular waves, the equation $u = \sqrt{gd_2} + \frac{3}{5}v_1$ gives results below, and sometimes much below, the observed velocities. The equation $u = \sqrt{gd_2} + v_1$ not only gives

better agreement with the author's experiments for individual groups or types of wave but gives results slightly higher than the observed wave velocities in three groups, and results slightly below in three groups, with an average for all the experiments nearly the same as the observed velocities.

Table 3. Comparison Between Observed Velocities and Velocities Computed by Bazin-Darcy Formulas (A) $u = \sqrt{gd_2} + \frac{3}{5}v_1$, (B) $u = \sqrt{gd_2} + v_1$

Type	Observed	Basin - Darcy			
		$\sqrt{gd_2} + \frac{3}{5}v_1$		$\sqrt{gd_2} + v_1$	
	u	u	Ratio $\frac{Col.3}{Col.2}$	u	Ratio $\frac{Col.5}{Col.2}$
(1)	(2)	(3)	(4)	(5)	(6)
Instantaneous Increase	4.69(a)	4.13	0.880	4.79	1.021
Instantaneous Decrease	5.13(b)	3.91	0.762	4.63	0.903
Gradual Increase	4.18(a)	4.11	0.983	4.47	1.069
Gradual Decrease	4.52(b)	3.96	0.876	4.48	0.993
Triangular-Stable depth	4.11(a)	4.24	1.032	4.57	1.110
-Crest depth	"	4.13	1.005	4.43	1.078
Rectangular	4.34(b)	3.65	0.841	4.02	0.925

(a) Velocity of toe of wave.
(b) Velocity of crest of wave.

In the tables subsequently given, containing the results of the author's individual experiments, velocities had been calculated by the first of the two above equations, i.e., adding $\frac{3}{5}v_1$ to $\sqrt{gd_2}$, before it was discovered that that was not the correct formula. A column was therefore added to the tables giving also the velocities calculated by the equation $u = \sqrt{gd_2} + v_1$.

Leach and King - An equation for velocity of an abrupt wave subject to momentum control was published by Horace W. King in "Civil Engineering" June 1933[2] and its derivation given. The same equation and its derivation was also given in a signed memorandum submitted to the author by H. R. Leach, dated January 5, 1927. Accordingly this wave formula has been designated as the Leach-King equation. Mr. Leach's derivation is as follows:

Referring to figure 9, assume wave moves from a to b in 1 second.

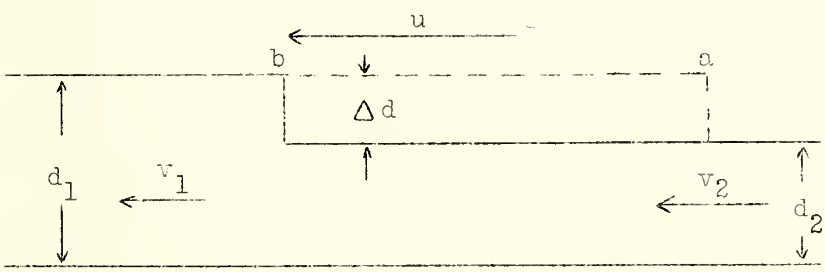

Fig. 9 - Decrement wave moving downstream.

Force = change of momentum per second = mass $(v_1 - v_2)$.

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[2] King, H. W. Translatory waves in open channels. Civil Engineering vol. 3, no. 6, pp. 319-321, June 1933.

The mass acted on in 1 second equals:

$$\text{mass} = w\frac{\left(d_2 u - d_2 v_2\right)}{g} = w\,\frac{d_2}{g}(u - v_2) \tag{1}$$

$$= w\,\frac{\left(d_1 u - d_1 v_1\right)}{g} = w\,\frac{d_1}{g}(u - v_1). \tag{2}$$

The force acting horizontally on the mass is the difference between the two horizontal end pressures;

$$\frac{d_1^2 - d_2^2}{2} = \frac{d_1}{g}(u - v_1)\ (v_1 - v_2). \tag{3}$$

Inflow -- Outflow = Change in volume;

$$v_2 d_2 - v_1 d_1 = u\Delta d = u(d_2 - d_1) \tag{4}$$

$$v_2 = \frac{u(d_2 - d_1) + v_1 d_1}{d_2} \tag{5}$$

or

$$v_1 = \frac{v_2 d_2 - u(d_2 - d_1)}{d_1} \tag{6}$$

Substituting equation (5) in equation (3),

$$\frac{d_1^2 - d_2^2}{2} = \frac{d_1}{g}(u - v_1)\left[v_1 - \frac{u(d_2 - d_1) + v_1 d_1}{d_2}\right] \tag{7}$$

$$= \frac{d_1}{g}(u - v_1)\ \left(\frac{v_1 d_2 - u d_2 + u d_1 - v_1 d_1}{d_2}\right)$$

$$= \frac{d_1}{g d_2}\ (u - v_1)\left[u\ (d_1 - d_2) - v_1(d_1 - d_2)\right]$$

$$\frac{d_1 + d_2}{2} = \frac{d_1}{g d_2}\ (u - v_1)^2 \tag{8}$$

$$(u - v_1)^2 = \frac{d_2}{d_1} \frac{d_1 + d_2}{2} g \tag{9}$$

$$u = \pm\sqrt{\frac{d_2}{d_1} \frac{d_1 + d_2}{2} g} + v_1 \tag{10}$$

If equation (6) instead of equation (5) is substituted, the derivation gives the alternative equation

$$u = \pm\sqrt{\frac{d_1}{d_2} \frac{d_1 + d_2}{2} g} + v_2 \tag{11}$$

The plus and minus sign is given in equations (10) and (11) for the reason that, by a similar process, the same equation is obtained for a wave traveling upstream as for one traveling downstream, but, in the latter case, the term under the radical is to be subtracted from the channel velocity.

Koch and Carstanjen - In 1926 Koch and Carstanjen[3] published a discussion of waves subject to momentum control, i.e., neglecting energy losses. The derivation of their equation for an increment wave traveling upstream follows the general lines given below. Velocities are considered positive in the direction of initial flow.

The following derivation is for the case of an increment wave traveling upstream. Referring to figure 9, the quantity of water Q_1 flows through an open channel of unit width and depth d_1 with a mean velocity v_1.

$$Q_1 = d_1 v_1. \tag{12}$$

[3] Koch and Carstanjen. Movement of waters and the accompanying forces (Ger.), Sec. VII, pp. 132-145; Julius Springer, 1926.

Through the sudden introduction of an obstruction, as by the partial closing of a gate, the discharge is reduced from Q_1 to Q_2. The velocity of approach is reduced from v_1 to v_2. A wave Δd starts upstream. The wave Δd travels upstream as an increment of the depth, with a velocity u. Upstream from the rise or increment wave, the discharge still remains equal to the original Q_1, with its velocity v_1 unchanged, but between the obstruction or gate and the increment wave the discharge is Q_2 and the velocity is reduced to v_2.

Consider the wave traveling upstream. For determination of u and Δd it is necessary to transform the moving increment wave into a standing wave by adding to all velocities the velocity -u, which equals but is opposite in direction to the velocity u of the increment wave. The velocity of the wave itself will then be zero and the problem can be viewed as representing a standing wave, in passing through which the stream depth changes from d_1 to $(d_1 + \Delta d) = d_2$. It may be assumed that for the small rise, Δd, there is no appreciable energy lost and that there is a smooth transition of depth from d_1 to d_2. Neglecting the small energy loss, the rise, Δd, can be determined from the laws of momentum and energy. From the equation of energy there follows:

$$d_1 + \frac{(v_1 + u)^2}{2g} = d_2 + \frac{(v_2 + u)^2}{2g}. \qquad (13)$$

Since the mass per unit length is proportional to the depth, the equation of continuity gives

$$d_1 (v_1 + u) = (d_1 + \Delta d)(v_2 + u) \qquad (14)$$

or

$$\Delta d = \frac{v_1 - v_2}{v_2 + u} d_1. \tag{15}$$

The energy equation can be written:

$$(v_1 + u)^2 - (v_2 + u)^2 + 2g (d_2 - d_1) = 2g \Delta d. \tag{16}$$

Substituting the value of Δd (eq. 4) and simplifying,

$$(v_1 - v_2) (v_1 + v_2 + 2u) = 2g \frac{v_1 - v_2}{v_2 + u} d_1 \tag{17}$$

or

$$2u^2 + u (v_1 + 3v_2) = 2gd_1 - (v_1 + v_2) v_2. \tag{18}$$

Completing and solving the quadratic,

$$u = - \frac{v_1 + 3v_2}{4} \pm \sqrt{gd_1 + \frac{(v_1 + 3v_2)^2}{16} - \frac{v_1 + v_2}{2} v_2}$$

$$= \frac{v_1 + 3v_2}{4} \pm \sqrt{gd_1 + \left(\frac{v_1 - v_2}{4}\right)^2}. \tag{19}$$

The second term under the radical, $\left(\frac{v_1 - v_2}{4}\right)^2$ is negligibly small relative to gd_1, so that, with sufficient accuracy,

$$u = - \frac{v_1 + 3v_2}{4} + \sqrt{gd_1}. \tag{20}$$

Similarly, Koch and Carstanjen obtained for a decrement wave traveling downstream,

$$u = \frac{v_1 + 3v_2}{4} + \sqrt{gd_1 + \frac{v_1 - v_2}{4}^2} \tag{21}$$

or, approximately,

$$u = \frac{v_1 + 3v_2}{4} + \sqrt{gd_1}. \tag{22}$$

For the case of an increment wave which has traveled up through a channel to a large pond or reservoir and then returns as an increment wave traveling downstream, Koch and Carstanjen's analysis leads to the equation

$$u = v_1 + \left(1 - \frac{d_2 - d_1}{4d_1}\right)\sqrt{gd_1}\ , \tag{23}$$

and for a decrement wave traveling upstream,

$$u = -v_1 + \sqrt{\frac{2}{1 + \frac{d_2}{d_1}}}\ \sqrt{gd_1}\ . \tag{24}$$

It will be seen that Koch and Carstanjen obtained an equation in general form applying to an increment wave traveling upstream, or a decrement wave traveling downstream, but obtained two quite different equations for increment waves traveling downstream, or decrement waves traveling upstream. A comparison was made between the Koch and Carstanjen formula

$$u = \pm\ \frac{v_1 + 3v_2}{4} \pm \sqrt{gd_1} \tag{25}$$

and the special formula (23), above given, for an increment wave traveling downstream, as applied to the author's experiments on increment waves traveling downstream. The results of this comparison are shown on table 4. It will be seen that the agreement of equation (22) with the experiments is better than that of the special equation (23) given by Koch and Carstanjen for this case. Consequently equation (22) has been assumed to be general in its application and has been used for the purpose of comparison with the author's experiments in all cases.

Table 4 - Comparison Between Observed Velocities and Velocities Computed by Koch-Carstanjen Formulas for Increment Waves Traveling Downstream.

Group	No. of Expts. in Group	Value of u By Formula (23)	Value of u By Formula (22)	Observed u
(1)	(2)	(3)	(4)	(5)
Series A-4 inch end weir	2	4.02	4.38	(a) 4.20
Series A-3 inch end weir	4	4.22	4.56	4.45
Series A-1½ inch end weir	4	3.85	4.26	4.01
Series D-1½ inch end weir	2	3.60	3.96	3.23
Series D- Open end	3	4.25	4.52	4.44
Series E- Open end	3	4.15	4.38	4.50
Series C- Open end	5	4.40	4.75	4.63
Series B- Open end	9	4.85	5.33	5.58
Series B'- Open end	2	3.98	4.49	4.95
Average		4.32	4.69	4.69

(a) Velocity of toe.

It may be noted that the Leach-King and Koch-Carstanjen formulas are derived from the same assumptions or premises, namely, neglect of friction and a consideration of the energy equation and the momentum equation or the equation of continuity, yet the resulting formulas are quite different in form and not readily reducible to identity.

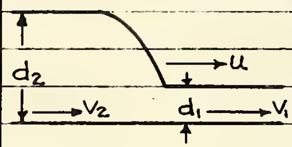
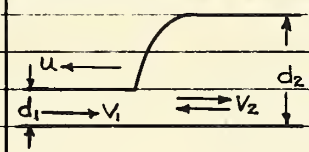
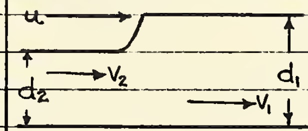
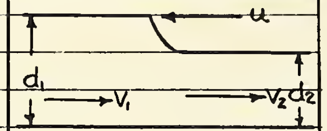
SUMMARY OF FORMULAS - Table 5 contains a summary of formulas used in comparing the results of the author's experiments with the calculated velocities for different types of waves. Inasmuch as only waves traveling in the same sense as the initial current were included in these experiments, the equations for waves traveling upstream have not been applied but are given for the sake of completeness.

RESULTS OF EXPERIMENTS - The results of the author's experiments for different types of waves, together with the velocities computed by the use of the formulas given on table 5, are shown on tables 6 to 11 inclusive. These results relate to experiments on waves in moving water. Other experiments on waves in still water are given in subsequent tables.

It should be noted that the velocities v_1 and v_2, given in column headings of Tables 6 to 11 inclusive, are those corresponding to stable uniform flow and have been obtained from the diagrams for velocities corresponding to the given depths and discharge rates (figs. 7 and 8).

In case of most types of waves it was found that either the toe or the crest was sharply defined, but not both. Accordingly in giving

TABLE 5 ~ WAVES SUBJECT TO MOMENTUM CONTROL ~ EQUATIONS OF COMPARISON WITH OBSERVED VELOCITIES

Type	Increment travelling Downstream	Increment travelling Upstream	Decrement travelling Downstream	Decrement travelling Upstream
Form				
Condition	Below dam Gates opened	Above dam Gates closed	Below dam Gates closed	Above dam Gates opened
Bazin and Darcy (A)	$u = \sqrt{gd_2} + \frac{3}{5} V_1$	$u = \sqrt{gd_2} - \frac{2}{5} V_1$	$u = \sqrt{gd_2} + \frac{3}{5} V_1$	$u = \sqrt{gd_2} - \frac{2}{5} V_1$
Bazin and Darcy (B)	$u = \sqrt{gd_2} + V_1$	$u = \sqrt{gd_2} - V_1$	$u = \sqrt{gd_2} + V_1$	$u = \sqrt{gd_2} - V_1$
Leach and King	$u = \sqrt{\frac{g}{2} \cdot \frac{d_2}{d_1}(d_2+d_1)} + V_1$	$u = \sqrt{\frac{g}{2} \cdot \frac{d_2}{d_1}(d_2+d_1)} - V_1$	$u = \sqrt{\frac{g}{2} \cdot \frac{d_2}{d_1}(d_2+d_1)} + V_1$	$u = \sqrt{\frac{g}{2} \cdot \frac{d_2}{d_1}(d_2+d_1)} - V_1$
	or $u = \sqrt{\frac{1+\rho}{2\rho}} \cdot \sqrt{gd_2} \pm V_1$	where $\rho = \frac{d_1}{d_2}$	$u = \sqrt{\frac{\phi(\phi+1)}{2}} \sqrt{gd_1} \pm V_1$ where $\phi = \frac{d_2}{d_1}$	
Koch and Carstanjen	$u = \sqrt{gd_1} + \frac{V_1 + 3V_2}{4}$	$u = \sqrt{gd_1} - \frac{V_1 + 3V_2}{4}$	$u = \sqrt{gd_1} + \frac{V_1 + 3V_2}{4}$	$u = \sqrt{gd_1} - \frac{V_1 + 3V_2}{4}$

(a) For waves with breaking crest

TABLE 6-a; CHANNEL WAVE EXPERIMENTS ~ INSTANTANEOUS INCREASE IN DISCHARGE

		Initial Condition				Ultimate Stable Condition											
Expt. No.	Slope of Flume S_c	Depth d_1 Inches	Discharge Q_1 c.f.s.	Velocity V_1 f.p.s.	$\Delta q = Q_2 - Q_1$ c.f.s.	Depth d_2 Inches	Discharge Q_2 c.f.s.	Velocity V_2 f.p.s.	Depth at Cope d_K Inches	Initial rise in stage $d_1 - d_K$ Inches	Observed Velocity of Toe U_t f.p.s.	$\sqrt{gd_1}$	$\sqrt{gd_2}$	Bazin-Darcy u Formula A f.p.s.	Leach-King u f.p.s.	Koch-Car't'jen u f.p.s.	Bazin-Darcy u Formula B f.p.s.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
			SERIES A ~ 4 INCH END WEIR						(a)	(a)				(c)	(c)	(c)	(c)
16	level	4.44	0.074	0.426	0.066	5.38	0.140	0.67 (b)	4.06	0.38	3.94	3.46	3.80	4.05	4.42	4.07	4.23
17	"	5.38	.140	.666	.098	6.18	.238	.98 (b)	5.83	.45	4.47	3.80	4.08	4.48	4.91	4.70	4.75
Average		4.91		0.546		5.78		0.82			4.20	3.63	3.94	4.26	4.66	4.38	4.49
			SERIES A ~ 3 INCH END WEIR														
18	level	4.85	0.075	0.396	0.070	5.71	0.145	0.64	5.32	0.47	3.86	3.61	3.92	4.16	4.50	4.19	4.32
19	"	5.69	.145	.651	.057	6.33	.202	.81	5.97	.28	4.54	3.91	4.13	4.52	4.90	4.68	4.78
20	"	6.30	.202	.820	.049	6.82	.251	.93	6.53	.23	5.16	4.12	4.28	4.77	5.19	5.02	5.10
25	"	4.88	.075	.393	.136	6.43	.211	.83	5.54	.66	4.24	3.62	4.16	4.40	4.87	4.10	4.55
Average		5.43		0.565		6.32		0.80			4.45	3.82	4.12	4.46	4.86	4.50	4.69
			SERIES A – 1½ INCH END WEIR														
55	level	3.43	0.074	0.552	0.068	4.22	0.142	0.86	3.74	0.31	3.34	3.03	3.37	3.70	4.11	3.81	3.92
56	"	4.25	.142	.854	.070	5.00	.212	1.09	4.62	.37	4.26	3.38	3.67	4.18	4.68	4.41	4.52
57	"	5.02	.212	1.080	.055	5.52	.267	1.24	5.30	.28	4.75	3.67	3.85	4.50	5.03	4.87	4.93
62	"	3.38	.074	.560	.134	4.95	.208	1.08	4.00	.62	3.70	3.01	3.65	3.99	4.60	3.96	4.21
Average		4.02		0.762		4.92		1.07			4.01	3.27	3.64	4.09	4.60	4.26	4.40
			SERIES D – 1.0 INCH END WEIR														
163	0.00102	2.01	0.064	0.814	0.066	3.01	0.130	1.13	2.53	0.52	3.48	2.32	2.85	3.34	3.99	3.37	3.66
165	"	3.75	.191	1.300	.054	4.42	.245	1.41	—	—	2.99	3.17	3.45	4.23	4.91	4.55	4.75
Average		2.88		1.06		3.72		1.27			3.23	2.74	3.15	3.78	4.45	3.96	4.20
			SERIES D – OPEN END														
175	0.00102	2.38	0.131	1.41	0.063	3.15	0.194	1.57	2.77	0.39	4.21	2.53	2.91	3.76	4.55	4.06	4.32
177	"	3.79	.248	1.67	.042	4.32	.290	1.72	4.09	.30	5.20	3.19	3.41	4.41	5.20	4.90	5.08
181	"	3.19	.194	1.56	.096	4.32	.290	1.72	3.70	.51	4.36	2.93	3.41	4.35	5.26	4.61	4.97
Average		3.12		1.55		3.93		1.67			4.44	2.88	3.24	4.17	5.00	4.52	4.79
			SERIES E – OPEN END														
189	0.00186	1.26	0.066	1.34	0.065	2.07	0.131	1.61	1.73	0.47	3.33	1.84	2.36	3.16	4.05	3.38	3.70
191	"	2.83	.193	1.74	.053	3.30	.246	1.92	3.19	.36	4.34	2.76	2.98	4.02	4.85	4.64	4.72
193	"	3.88	.291	1.91	.034	4.33	.325	1.81	4.28	.40	5.82	3.30	3.41	4.56	5.42	5.13	5.32
Average		2.66		1.66		3.23		1.78			4.50	2.63	2.92	3.91	4.77	4.38	4.58

TABLE 6-b, CHANNEL WAVE EXPERIMENTS ~ INSTANTANEOUS INCREASE IN DISCHARGE

		Initial Condition				Ultimate Stable Condition											
Expt. No.	Slope of Flume S_c	Depth d_1 Inches	Discharge Q_1 c.f.s.	Velocity V_1 f.p.s.	$\Delta q = Q_2 - Q_1$ c.f.s.	Depth d_2 Inches	Discharge Q_2 c.f.s.	Velocity V_2 f.p.s.	Depth at Cope d_K Inches	Initial rise in stage $d_1 - d_K$ Inches	Velocity of Toe U_t (observed) f.p.s.	$\sqrt{gd_1}$	$\sqrt{gd_2}$	Bazin-Darcy u Formula A f.p.s.	Leach-King u f.p.s.	Koch-Car't'jen u f.p.s.	Bazin-Darcy u Formula B f.p.s.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
			SERIES C – OPEN END						(a)	(a)				(c)	(c)	(c)	(c)
124	0.00392	1.25	0.068	1.43	0.077	2.03	0.145	1.83	2.11	0.86	3.32	1.83	2.34	3.44	4.51	3.62	3.77
125	"	1.99	.138	1.78	.074	2.60	.212	2.08	2.68	.69	4.34	2.32	2.64	3.71	4.61	4.33	4.42
126	"	2.60	.210	2.06	.068	3.10	.278	2.28	3.19	.59	5.10	2.64	2.89	4.13	5.09	4.86	4.95
127	"	3.11	.278	2.29	.067	3.60	.345	2.45	3.70	.59	5.88	2.89	3.11	4.48	5.51	5.30	5.40
128	"	3.60	.345	2.45	.067	4.06	.412	2.60	4.31	.71	4.65	3.11	3.30	4.77	5.86	5.67	5.75
Average		2.51		2.08		3.13		2.25			4.66	2.56	2.86	4.11	5.12	4.76	4.86
			SERIES B – OPEN END														
84	0.00736	1.05	0.074	1.80	0.068	1.56	0.142	2.36	1.51	0.46	4.00	1.68	2.05	3.13	4.08	3.90	3.85
85	"	1.54	.142	2.36	.069	2.00	.211	2.70	2.00	.46	5.00	2.04	2.32	3.74	4.85	4.66	4.68
86	"	2.00	.211	2.70	.063	2.36	.274	2.98	2.36	.36	5.89	2.32	2.52	4.14	5.34	5.23	5.22
87	"	2.39	.274	2.94	.072	2.75	.346	3.23	2.77	.38	6.03	2.54	2.72	4.48	5.77	5.70	5.66
94	"	2.64	.348	3.36	.080	3.17	.428	3.48	3.16	.52	6.67	2.66	2.92	4.93	6.43	6.11	6.28
100	"	1.56	.138	2.26	.138	2.36	.276	2.98	2.24	.68	4.69	2.05	2.52	3.88	5.08	4.85	4.78
101	"	2.36	.276	2.99	.125	3.02	.401	3.39	2.87	.51	5.98	2.52	2.85	4.64	6.03	5.81	5.84
88	"	2.79	.348	3.19	.114	3.33	.462	3.56	3.26	.47	6.58	2.74	2.99	4.90	6.33	6.21	6.18
107	"	2.03	.210	2.64	.180	2.97	.390	3.36	2.83	.80	5.37	2.33	2.83	4.41	5.78	5.51	5.47
Average		2.04		2.69		2.61		3.12			5.58	2.32	2.64	4.25	5.52	5.33	5.33
			SERIES B′ – OPEN END														
111	0.00790	1.05	0.069	1.68	0.073	1.60	0.142	2.26	1.34	0.29	4.06	1.68	2.07	3.08	4.00	3.79	3.75
113	"	2.04	.207	2.59	.078	2.47	.285	2.94	2.48	.44	5.84	2.34	2.58	4.13	5.31	5.19	5.17
Average		1.54		2.13		2.03		2.60			4.95	2.01	2.32	3.60	4.65	4.49	4.46
Average of 34		3.08		1.68		3.78		1.96			4.69	2.80	3.13	4.13	5.00	4.68	4.79

(a) Average of gages 2,3 and 4

(b) 4" weir not rated. Ultimate stage uncertain.

(c) Computed with reference to stable flow, using V_1, V_2, d_1 and d_2

TABLE 7-a, CHANNEL WAVE EXPERIMENTS ~ INSTANTANEOUS DECREASE IN DISCHARGE

		Initial Condition				Ultimate Stable Condition											
Expt. No.	Slope of Flume S_c	Depth d_1 Inches	Discharge Q_1 c.f.s	Velocity V_1 f.p.s.	$\Delta q = Q_1 - Q_2$ c.f.s.	Depth d_2 Inches	Discharge Q_2 c.f.s.	Velocity V_2 f.p.s.	Depth at Heel d_h Inches	Initial Drop in Stage $d_1 - d_h$ Inches	Observed Velocity of Crest u_c f.p.s.	$\sqrt{gd_1}$	$\sqrt{gd_2}$	Bazin-Darcy u Formula A f.p.s.	Leach-King u f.p.s.	Koch-Car't'jen u f.p.s.	Bazin-Darcy u Formula B f.p.s.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
			SERIES A ~ 3 INCH END WEIR						(a)	(a)				(c)	(c)	(c)	(c)
23	level	5.71	0.145	0.649	0.078	4.73	0.067	0.38	5.32	0.39	4.43	3.92	3.57	3.96	4.06	4.37	4.22
22	"	6.06	.175	.739	.030	5.71	.145	.64	5.92	.14	4.58	4.04	3.92	4.36	4.60	4.71	4.66
21	"	6.79	.245	.923	.070	6.06	.175	.73	6.42	.37	4.92	4.27	4.04	4.59	4.85	5.05	4.96
26	"	6.48	.210	.829	.118	5.06	.092	.47	5.95	.53	4.83	4.17	3.69	4.19	4.31	4.73	4.52
Average		6.26		0.785		5.39		0.55			4.69	4.10	3.80	4.28	4.45	4.72	4.59
			SERIES A ~ $1\frac{1}{2}$ INCH END WEIR														
61	level	3.32	0.068	0.52	0.068	1.50	0	0	2.86	0.46	3.39	2.99	2.01	2.32	2.24	3.12	2.53
60	"	4.23	.138	.83	.070	3.22	0.060	0.56	3.87	.37	4.03	3.37	2.94	3.44	3.59	4.00	3.77
59	"	5.02	.205	1.04	.067	4.18	.138	.85	4.69	.33	4.46	3.67	3.35	3.97	4.25	4.57	4.39
58	"	5.63	.267	1.27	.062	4.93	.205	1.07	5.39	.24	4.73	3.89	3.64	4.40	4.47	5.01	4.91
63	"	4.92	.208	1.08	.134	3.47	.074	.55	4.27	.65	4.56	3.64	3.06	3.71	3.90	4.32	4.14
Average		4.62		0.95		3.46		0.61			4.23	3.51	3.00	3.57	3.69	4.20	3.95
			SERIES D ~ 1.0 INCH END WEIR														
164	0.00102	3.01	0.130	1.11	0.066	2.00	0.064	0.81	2.89	0.12	4.11 (b)	2.79	2.32	2.99	3.19	3.68	3.43
166	"	3.95	.206	1.33	.044	3.40	.162	1.22	3.83	.12	4.55 (b)	3.26	3.02	3.82	4.25	4.51	4.35
Average		3.48		1.22		2.70		1.02			4.33	3.02	2.67	3.40	3.72	4.10	3.89
			SERIES D ~ OPEN END														
176	0.00102	3.26	0.194	1.52	0.063	2.36	0.131	1.42	2.91	0.35	5.10	2.96	2.52	3.43	3.86	4.41	4.04
178	"	4.31	.290	1.72	.042	3.80	.248	1.66	4.11	.20	6.03	3.40	3.20	4.23	4.82	5.08	4.92
182	"	4.30	.290	1.73	.096	3.15	.194	1.57	3.63	.67	5.18	3.40	2.91	3.95	4.45	5.01	4.64
Average		3.96		1.66		3.10		1.55			5.44	3.25	2.88	3.87	4.38	4.83	4.53
			SERIES E ~ OPEN END														
190	0.00186	2.11	0.131	1.59	0.065	1.28	0.066	1.31	1.79	0.32	3.45	2.38	1.85	2.80	3.26	3.76	3.44
192	"	3.42	.246	1.85	.053	2.76	.193	1.78	3.20	.22	5.16	3.03	2.72	3.83	4.44	4.83	4.57
194	"	4.33	.306	1.81	.015	3.73	.291	2.04	4.19	.14	5.44	3.41	3.17	4.26	4.88	5.39	4.98
Average		3.29		1.75		2.59		1.71			4.68	2.94	2.58	3.63	4.19	4.66	4.33

TABLE 7-b, CHANNEL WAVE EXPERIMENTS ~ INSTANTANEOUS DECREASE IN DISCHARGE

		Initial Condition				Ultimate Stable Condition											
Expt. No.	Slope of Flume S_c	Depth d_1 Inches	Discharge Q_1 c.f.s.	Velocity V_1 f.p.s.	$\Delta q = Q_1 - Q_2$ c.f.s.	Depth d_2 Inches	Discharge Q_2 c.f.s.	Velocity V_2 f.p.s.	Depth at Heel d_h Inches	Initial Drop in Stage $d_1 - d_h$ Inches	Observed Velocity of Crest u_c f.p.s.	$\sqrt{gd_1}$	$\sqrt{gd_2}$	Bazin-Darcy u Formula A f.p.s.	Leach-King u f.p.s.	Koch-Car't'jen u f.p.s.	Bazin-Darcy u Formula B f.p.s.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
			SERIES C ~ OPEN END						(a)	(a)				(c)	(c)	(c)	(c)
133	0.00392	1.94	0.138	1.82	0.070	1.25	0.068	1.42	1.25	0.76	4.67	2.28	1.83	2.92	3.48	3.80	3.65
132	"	2.56	.210	2.09	.072	1.97	.138	1.80	2.00	.63	4.35	2.62	2.30	3.55	4.25	4.49	4.39
130	"	3.58	.345	2.39	.072	3.05	.273	2.26	3.14	.51	6.05	3.10	2.86	4.29	5.15	5.39	5.25
129	"	4.07	.410	2.57	.072	3.55	.338	2.44	3.61	.55	6.17	3.31	3.09	4.63	5.57	5.78	5.66
Average		3.05		2.22		2.45		1.98			5.31	2.83	2.52	3.85	4.61	4.86	4.74
			SERIES B ~ OPEN END														
91	0.00736	2.05	0.218	2.72	0.026	1.87	0.192	2.62	1.88	0.18	5.37	2.35	2.24	3.87	4.92	4.99	4.96
97	"	1.60	.148	2.36	.076	1.00	.072	1.85	1.06	.51	5.00	2.07	1.64	3.06	3.84	4.05	4.00
92	"	1.82	.190	2.67	.082	1.30	.108	2.13	1.33	.54	4.69	2.21	1.87	3.47	4.41	4.48	4.54
96	"	2.04	.212	2.66	.070	1.56	.142	2.36	1.58	.44	5.43	2.34	2.05	3.65	4.59	4.77	4.71
95	"	2.61	.347	3.39	.069	2.38	.278	3.00	2.36	.37	6.35	2.65	2.53	4.56	5.87	5.75	5.92
89	"	3.01	.406	3.45	.056	2.76	.350	3.24	2.74	.31	6.35	2.84	2.72	4.79	6.11	6.13	6.17
103	"	2.39	.275	2.95	.135	1.54	.140	2.33	1.57	.83	5.84	2.54	2.04	3.81	4.80	5.02	4.99
90	"	2.62	.348	3.40	.131	2.02	.217	2.74	2.03	.74	5.85	2.65	2.33	4.37	5.59	5.55	5.73
102	"	3.06	.401	3.35	.126	2.38	.275	2.98	2.40	.70	6.18	2.87	2.53	4.54	5.74	5.94	5.88
108	"	3.06	.390	3.26	.182	1.97	.208	2.70	2.03	1.07	6.27	2.87	2.30	4.26	5.35	5.71	5.56
Average		2.43		3.02		1.88		2.60			5.73	2.54	2.22	4.04	5.12	5.24	5.25
			SERIES B′ ~ OPEN END														
112	0.00790	1.58	0.138	2.23	0.071	0.97	0.067	1.68	1.06	0.55	4.69	2.06	1.53	2.87	3.68	3.88	3.76
114	"	2.43	.276	2.91	.070	2.02	.206	2.59	1.96	.49	7.13	2.56	2.33	4.08	5.15	5.23	5.24
Average		2.00		2.57		1.50		2.14			5.91	2.31	1.93	3.48	4.42	4.56	4.50
Average of 33		3.55		1.93		2.82		1.67			5.13	3.03	2.67	3.85	4.48	4.77	4.63

(a) Average of gages 2, 3 and 4

(b) Erratic runs

(c) Computed with reference to stable flow, using V_1, V_2, d_1 and d_2

TABLE 8 – CHANNEL WAVE EXPERIMENTS – GRADUAL INCREASE IN DISCHARGE

Expt. No.	Slope of Flume S_0	Initial Condition			Increase in Discharge			Ultimate Stable Condition			Observed Velocity of Toe u_t	$\sqrt{gd_1}$	$\sqrt{gd_2}$	Bazin-Darcy u Formula A	Leach-King u	Koch-Car'tjen u	Bazin-Darcy u Formula B
		Depth d_1	Discharge Q_1	Velocity V_1	$\Delta q = Q_2 - Q_1$	Duration of Increase	Rate of Increase	Depth d_2	Discharge Q_2	Velocity V_2							
		Inches	c.f.s.	f.p.s.	c.f.s.	Secs	c.f.s./Sec.	Inches	c.f.s.	f.p.s.	f.p.s.			f.p.s.	f.p.s.	f.p.s.	f.p.s.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
		SERIES A – 3 INCH END WEIR												(b)	(b)	(b)	(b)
34	level	3.42	0	0	0.070	120	0.00058	4.75	0.070	0.400	3.14	3.03	3.57	3.57	3.90	3.33	3.57
37	"	4.90	0.073	0.381	.073	12	.00608	5.72	.146	.640	3.54	3.63	3.92	4.15	4.47	4.21	4.30
36	"	4.90	.075	.391	.085	15	.00566	5.88	.160	.690	3.65	3.63	3.98	4.22	4.57	4.25	4.37
27	"	4.87	.074	.388	.069	25	.00276	5.70	.143	.640	4.80	3.62	3.91	4.14	4.47	4.20	4.30
35	"	4.80	.070	.373	.070	120	.00058	5.65	.140	.630	3.51	3.59	3.90	4.12	4.45	4.16	4.27
38	"	5.78	.146	.646	.069	12	.00575	6.48	.215	.840	3.98	3.94	4.17	4.56	4.95	4.73	4.82
28	"	5.68	.143	.644	.070	25	.0028	6.45	.213	.838	4.80	3.91	4.16	4.55	4.94	4.70	4.80
39	"	6.48	.215	.848	.059	12	.00492	7.05	.274	1.00	4.85	4.17	4.35	4.86	5.30	5.13	5.20
29	"	6.44	.213	.846	.065	25	.0026	7.10	.278	1.03	5.47	4.16	4.37	4.88	5.33	5.14	5.22
Average		5.25		.502				6.09		0.745	4.19	3.74	4.04	4.34	4.71	4.43	4.54
		SERIES A – 1½ INCH END WEIR															
64	level	3.35	0.075	0.572	0.071	12	0.006	4.26	0.146	0.880	3.63	3.00	3.38	3.72	4.17	3.80	3.95
71	"	3.35	.074	.565	.072	24	.003	4.25	.146	.880	3.39	3.00	3.37	3.71	4.17	3.80	3.94
75	"	3.39	.075	.566	.070	48	.0014	4.27	.145	.880	3.40	3.02	3.39	3.73	4.17	3.82	3.96
65	"	4.27	.146	.874	.064	12	.0053	4.95	.210	1.08	4.21	3.39	3.65	4.17	4.66	4.42	4.52
66	"	4.97	.210	1.080	.061	12	.0051	5.56	.271	1.25	4.17	3.65	3.87	4.52	5.06	4.86	4.95
73	"	5.03	.215	1.10	.062	24	.0026	5.61	.277	1.26	4.02	3.68	3.88	4.54	5.09	4.90	4.98
77	"	4.99	.213	1.09	.063	48	.0013	5.62	.276	1.26	3.94	3.66	3.89	4.54	5.10	4.88	4.98
Average		4.18		0.835				4.93		1.07	3.82	3.34	3.63	4.13	4.63	4.35	4.47
		SERIES B′ – OPEN END															
115	0.0079	1.60	0.139	2.22	0.069	15	0.0046	2.05	0.208	2.61	5.25	2.07	2.35	3.68	4.73	4.58	4.57
117	"	2.35	.278	3.03	.067	12	.00558	2.80	.345	3.18	6.35	2.51	2.74	4.56	5.90	5.65	5.77
116	"	0	0	0	.137	26	.00527	1.60	.137	2.25	2.46	0	2.07	2.07	(a)	1.69	2.07
118	"	1.52	.138	2.32	.139	24	.00579	2.45	.277	3.20	5.11	2.02	2.57	3.96	5.25	5.00	4.89
Average		1.37		1.89				2.22		2.81	4.79	1.65	2.43	3.56	5.29	4.23	4.32
Average of 20		4.10		0.897				4.91		1.27	4.18	3.18	3.57	4.11	4.77	4.36	4.47

(a) Formula does not apply (b) Computed with reference to stable flow, using V_1, V_2, d_1 and d_2

TABLE 9 – CHANNEL WAVE EXPERIMENTS – GRADUAL DECREASE IN DISCHARGE

Expt. No.	Slope of Flume S_0	Initial Condition			Decrease in Discharge			Ultimate Stable Condition			Observed Velocity of Crest u_c	$\sqrt{gd_1}$	$\sqrt{gd_2}$	Bazin-Darcy u Formula A	Leach-King u	Koch-Car'tjen u	Bazin-Darcy u Formula B
		Depth d_1	Discharge Q_1	Velocity V_1	$\Delta q = Q_2 - Q_1$	Duration of Decrease	Rate of Decrease	Depth d_2	Discharge Q_2	Velocity V_2							
		Inches	c.f.s.	f.p.s.	c.f.s.	Secs.	c.f.s./Sec.	Inches	c.f.s.	f.p.s.	f.p.s.			f.p.s.	f.p.s.	f.p.s.	f.p.s.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
		SERIES A ~ 3 INCH END WEIR												(a)	(a)	(a)	(a)
43	level	4.92	0.074	0.385	0.074	12	0.00617	3.00	0	0	3.71	3.64	2.84	3.07	2.94	3.73	3.22
33	"	4.92	.073	.380	.073	25	.00290	3.00	0	0	4.15	3.64	2.84	3.07	2.93	3.73	3.22
42	"	5.77	.147	.651	.073	12	.00610	4.78	0.074	0.41	4.65	3.94	3.58	3.97	4.08	4.41	4.23
32	"	5.82	.145	.640	.072	25	.00288	4.78	.073	.41	4.31	3.96	3.58	3.96	4.07	4.43	4.22
41	"	6.47	.214	.846	.067	12	.00580	5.70	.147	.64	4.59	4.17	3.91	4.42	4.65	4.86	4.76
31	"	6.56	.215	.840	.070	25	.00280	5.60	.145	.64	5.08	4.20	3.88	4.38	4.58	4.89	4.72
40	"	7.09	.275	.992	.061	12	.00510	6.45	.214	.84	4.48	4.37	4.16	4.76	5.07	5.25	5.15
30	"	7.12	.273	.990	.061	25	.00240	6.47	.215	.84	5.01	4.38	4.17	4.76	5.06	5.26	5.16
Average		6.08		0.716				4.72		0.47	4.50	4.04	3.62	4.05	4.17	4.57	4.34
		SERIES A – 1½ INCH END WEIR															
70	level	3.50	0.083	0.606	0.083	15	0.0055	1.50	0	0	4.00	3.07	2.01	2.37	1.72	3.22	2.62
69	"	4.36	.155	.909	.072	12	.0060	3.45	0.083	0.63	4.21	3.42	3.05	3.59	3.79	4.12	3.96
72	"	4.28	.146	.872	.060	24	.0025	3.50	.086	.65	3.41	3.39	3.07	3.59	3.79	4.09	3.94
76	"	4.27	.145	.868	.069	46	.0015	3.37	.076	.60	3.73	3.39	3.01	3.53	3.72	4.06	3.88
68	"	5.06	.218	1.08	.063	12	.0053	4.40	.155	.92	4.44	3.69	3.44	4.09	4.41	4.65	4.52
67	"	5.56	.270	1.24	.052	12	.0043	5.05	.218	1.10	4.29	3.87	3.68	4.42	4.84	5.00	4.94
74	"	5.62	.277	1.26	.055	24	.0023	5.62	.222	1.26	3.77	3.89	3.89	4.64	5.15	5.15	5.15
78	"	5.63	.276	1.25	.061	49	.0013	5.02	.215	1.10	4.55	3.89	3.67	4.42	4.82	5.03	4.92
Average		4.41		0.935				3.86		0.78	4.05	3.58	3.23	3.83	4.03	4.42	4.24
		SERIES B′ – OPEN END															
122	0.0079	2.40	0.277	2.95	0.070	12	0.00583	2.05	0.207	2.50	4.92	2.54	2.35	4.12	5.21	5.15	5.30
120	"	2.38	.278	2.99	.208	36	.00578	1.00	.070	1.71	4.96	2.53	1.64	3.43	4.38	4.56	4.63
121	"	3.07	.405	3.38	.059	12	.00492	2.80	.346	3.18	6.33	2.87	2.74	4.77	6.06	6.10	6.12
119	"	2.40	.275	2.93	.135	24	.00562	1.60	.140	2.24	5.79	2.54	2.07	3.83	4.82	4.95	5.00
Average		2.56		3.08				1.86		2.41	5.50	2.61	2.20	4.04	5.09	5.15	5.26
Average of 20		4.86		1.30				3.96		0.98	4.52	3.57	3.18	3.96	4.30	4.63	4.48

(a) Computed with reference to stable flow, using V_1, V_2, d_1, and d_2

TABLE 10 ~ CHANNEL WAVE EXPERIMENTS ~ TRIANGULAR WAVES

UNIFORM RATE OF INCREASE OF DISCHARGE TO A GIVEN PEAK THENCE DECREASING AT UNIFORM RATE TO INITIAL FLOW

Expt. No.	Slope of Flume S_c	Initial Condition: Depth d_1 Inches	Initial Condition: Discharge Q_1 c.f.s.	Initial Condition: Velocity v_1 c.f.s.	Increase and Decrease in Discharge: Change in Discharge Δq c.f.s.	Duration of Increase and Decrease Secs.	Rate of Change $\Delta q/\Delta t$	Peak Discharge Q_2 c.f.s.	Duration of Peak Secs.	Stable depth for Q_2 (from rating curve) d_2 Inches	Velocity for stable Depth v_2 f.p.s.	Depth at Crest d_c Inches	Velocity for Crest depth v_2' f.p.s.	Height of Wave d_c-d_1 Inches
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
			SERIES A~3 INCH END WEIR		(b)	(b)	(b)					(a)		(a)
44	Level	4.87	0.070	0.368	0.067	12	0.0056	0.138	3	5.65	0.625	5.26	0.525	0.39
47	"	4.88	.076	.367	.071	23.5	.0031	.145	3	5.72	.640	5.32	.540	.44
49	"	6.12	.182	.761	.065	12	.0054	.247	4	6.80	.930	6.39	.818	.27
50	"	6.15	.182	.799	.127	24	.0053	.307	3	7.15	1.020	6.75	.915	.60
45	"	6.48	.212	.217	.055-.040	12-15	.0046 / .0027	.2675	3	7.01	.985	6.78	.925	.30
46	"	6.50	.210	.216	.055	24-28	.0023 / .0020	.265	4	6.99	.982	6.82	.930	32
48	"	4.90	.075	.391	.138	24-24.5	.0058	.213	4	6.46	.838	5.59	.610	.69
Average		5.70		0.445		19.32	0.0044		3.43	6.54	0.860	6.13	0.752	0.43
			SERIES A~1½ INCH END WEIR											
52	Level	3.51	0.078	0.568	0.072	12	0.0060	0.150	3	4.32	0.890	3.86	0.755	0.35
80	"	3.44	.078	.580	.072-.066	12-24	.0060 / .0027	.150	4	4.32	.890	3.83	.750	39
79	"	3.41	.075	.563	.073	24	.00304	.148	5	4.30	.885	3.81	.742	.40
54	"	4.68	.178	.972	.065-.068	12-11.5	.0054 / .0059	.243	3	5.30	1.170	4.98	1.085	.30
82	"	5.11	.218	1.09	.056-.054	12-23.5	.0047 / .0023	.274	4.5	5.58	1.257	5.31	1.175	.20
81	"	5.04	.215	1.09	.057-.055	24	.0024 / .0023	.272	5	5.55	1.246	5.38	1.196	.34
51	"	3.45	.075	.556	.138-.135	23-24	.0060 / .0056	.213	3	5.00	1.090	4.19	.855	.74
53	"	4.70	.180	.979	.125-.127	24-23.5	.0052 / .0054	.305	3	5.87	1.335	5.33	1.180	.63
Average		4.17		0.800		19.34	0.0045		3.81	5.03	1.095	4.59	0.967	0.42
Average of 15		4.88		.634		19.33	0.00445		3.63	5.73	0.986	5.31	0.867	0.43

Expt. No.	Observed Velocity of Toe u_t f.p.s.	$\sqrt{gd_1}$	$\sqrt{gd_2}$	$\sqrt{gd_c}$	Bazin-Darcy u (Based on Stable Depth) f.p.s.	Bazin-Darcy u (Based on Crest Depth) f.p.s.	Leach-King u (Based on Stable Depth) f.p.s.	Leach-King u (Based on Crest Depth) f.p.s.	Koch-Carstanjen u (Based on Stable Depth) f.p.s.	Koch-Carstanjen u (Based on Crest Depth) f.p.s.	Forchheimer u (Based on Stable Depth) f.p.s.	Forchheimer u (Based on Crest Depth) f.p.s.	Bazin-Darcy u (Based on Stable depth) f.p.s.	Bazin-Darcy u (Based on Crest depth) f.p.s.
(1)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)
					(c) (e)	(c) (f)	(e)	(f)	(e)	(f)	(e)	(f)	(d) (e)	(d) (f)
44	4.07	3.62	3.90	3.76	4.06	3.96	4.28	4.15	4.12	4.07	2.27	4.54	4.26	4.15
47	3.91	3.62	3.92	3.78	4.07	3.97	4.30	4.17	4.12	4.08	2.10	4.01	4.28	4.16
49	4.15	4.06	4.28	4.14	4.68	4.57	5.02	4.89	4.90	4.85	2.44	6.14	5.02	4.89
50	4.37	4.07	4.39	4.26	4.78	4.68	5.16	5.03	4.98	4.93	3.20	5.32	5.14	5.02
45	5.57	4.17	4.34	4.27	4.62	4.56	4.88	5.30	4.77	4.74	2.73	4.79	4.85	4.19
46	4.58	4.18	4.33	4.28	4.62	4.58	4.85	5.32	4.78	4.75	2.86	4.40	4.85	4.80
48	3.78	3.63	4.17	3.88	4.27	4.06	4.57	4.28	4.24	4.14	2.27	5.11	4.52	4.26
Average	4.35	3.91	4.19	4.05	4.47	4.34	4.72	4.73	4.56	4.51	2.55	4.90	4.70	4.58
52	3.25	3.07	3.41	3.22	3.68	3.54	4.00	3.81	3.80	3.73	2.28	5.26	3.97	3.80
80	3.73	3.04	3.41	3.21	3.67	3.52	3.99	3.80	3.78	3.70	2.09	4.74	3.96	3.79
79	3.78	3.03	3.40	3.20	3.65	3.51	3.97	3.78	3.75	3.68	2.10	4.68	3.94	3.76
54	4.35	3.55	3.78	3.66	4.30	4.22	4.75	4.63	4.62	4.58	2.69	5.56	4.74	4.63
82	4.66	3.71	3.87	3.78	4.49	4.42	4.97	4.88	4.88	4.84	3.04	7.19	4.96	4.88
81	3.85	3.68	3.86	3.80	4.47	4.42	4.95	4.88	4.85	4.82	2.86	4.29	4.94	4.88
51	3.60	3.05	3.67	3.36	3.85	3.62	4.26	3.84	3.88	3.76	2.28	4.76	4.18	3.92
53	4.06	3.55	3.97	3.78	4.46	4.32	4.95	4.76	4.71	4.63	2.71	5.08	4.92	4.75
Average	3.91	3.34	3.67	3.50	4.07	3.95	4.48	4.30	4.28	4.22	2.51	5.20	4.45	4.30
Average of 15	4.11	3.60	3.91	3.77	4.24	4.13	4.59	4.50	4.41	4.35	2.53	5.05	4.57	4.43

(a) Average of gages 2, 3 and 4

(b) Amount, duration and rate of decrease given by second figure where decrease does not equal increase.

(c) Computed by formula A (Table 5) (d) Computed by formula B (Table 5)

(e) Computed with reference to stable flow, using v_1, v_2, d_1 and d_2 (f) Computed with reference to crest depth, using v_1, v_2', d_1 and d_c

TABLE 11 – CHANNEL WAVE EXPERIMENTS – RECTANGULAR WAVES

INSTANTANEOUS INCREASE MAINTAINED FOR GIVEN PERIOD WITH INSTANTANEOUS DECREASE TO ORIGINAL FLOW

Expt. No.	Slope of Flume	Initial and Ultimate condition: Depth d_1 Inches	Discharge Q_1 c.f.s.	Velocity V_1 f.p.s.	$\Delta q = Q_2 - Q_1$ c.f.s.	Increased Flow Q_2 c.f.s.	Duration of Flow Q_2 Secs.	Front Cope of Wave: Depth d_K Inches	Δd $(d_K - d_1)$ Inches	Crest of Wave: Depth d_c Inches	Total Increase in depth Inches	Velocity V_2 f.p.s.	Observed Velocity of Wave: Front u f.p.s.	Crest u_c f.p.s.	$\sqrt{gd_1}$	$\sqrt{gd_2}$	Bazin-Darcy u Formula A f.p.s.	Leach-King u f.p.s.	Koch-Car'tjen u f.p.s.	Forch-heimer u f.p.s.	Bazin-Darcy u Formula B f.p.s.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
		(b)	SERIES A – OPEN END					(a)	(a)	(a)	(a)		(f)	(d)			(g)	(g)	(g)	(g)	(g)
135	level	2.11	0.051	0.618	0.054	0.105	5	2.65	0.44	2.55	0.43	0.388	3.27	3.60	2.39	2.62	2.92	3.02	2.89	3.14	3.20
136	"	2.02	.051	.646	.054	.105	10	2.40(c)	.38(c)	2.50(c)	.48(c)	.381	2.54	3.04	2.33	2.59	2.88	3.00	2.84	2.88	3.16
141	"	2.12	.051	.615	.054	.105	15	2.45	.33	2.55	.42	.388	2.84	3.61	2.39	2.62	2.92	3.02	2.89	3.22	3.20
142	"	2.17	.051	.601	.054	.105	20	2.43	.26	2.61	.42	.395	2.96	3.26	2.42	2.65	2.95	3.04	2.92	3.14	3.23
138	"	3.12	.160	1.31	.050	.210	10	3.44	.32	3.54	.41	.510	3.56	3.41	2.90	3.09	3.70	3.92	3.81	3.05	4.06
137	"	3.26	.160	1.26	.050	.210	15	3.43	.17	3.66	.38	.521	4.25	4.40	2.96	3.14	3.74	3.96	3.86	3.20	4.20
144	"	3.75	.160	1.09	.050	.210	20	4.00	.25	4.15	.34	.573	4.25	4.43	3.18	3.34	3.93	4.10	4.01	3.20	4.38
143	"	3.82	.160	1.07	.050	.210	24	4.07	.25	4.23	.37	.581	4.73	4.28	3.20	3.37	3.96	4.12	4.02	3.10	4.40
140	"	4.94	.255	1.32	.038	.293	10	5.15	.21	5.21	.22	.659	5.45	5.10	3.64	3.74	4.48	4.68	4.63	3.58	5.01
Average		3.03		0.948		0.173		3.32		3.44		0.488	3.76	3.90	2.82	3.02	3.50	3.65	3.54	3.17	3.87
			SERIES A – 1½ INCH END WEIR											(e)							
145	level	2.65	0.065	0.627	0.065	0.130	5			2.92	0.25	0.785	3.45	3.59	2.67	2.80	3.16	3.44	3.38	6.13	3.39
146	"	2.68	.065	.620	.065	.130	10			3.11	.39	.742	3.38	3.55	2.68	2.89	3.20	3.48	3.36	3.87	3.46
149	"	2.77	.065	.600	.065	.130	15			3.18	.42	.764	3.40	4.26	2.73	2.92	3.23	3.51	3.41	4.04	3.50
150	"	2.71	.065	.613	.065	.130	20			3.20	.49	.770	3.66	3.83	2.70	2.93	3.23	3.51	3.39	3.39	3.50
147	"	4.47	.193	1.11	.055	.248	5			4.65	.27	1.175	4.96	5.47	3.47	3.54	4.20	4.65	4.61	7.87	4.65
148	"	4.42	.193	1.12	.055	.248	10			4.73	.24	1.195	4.47	5.79	3.45	3.57	4.20	4.68	4.61	4.55	4.67
151	"	4.54	.193	1.09	.055	.248	15			4.83	.26	1.224	5.39	6.04	3.49	3.60	4.23	4.71	4.64	4.82	4.70
152	"	4.51	.193	1.09	.055	.248	20			4.87	.31	1.232	4.88	5.25	3.48	3.62	4.24	4.72	4.64	3.90	4.71
Average		3.59		0.859		0.189				3.94		0.986	4.20	4.72	3.08	3.23	3.71	4.09	4.00	4.82	4.07
			SERIES A – 3 INCH END WEIR																		
153	level	3.92	0.064	0.417	0.066	0.130	5			4.24	0.32	0.502	4.06	4.03	3.25	3.38	3.59	3.78	3.71	5.28	3.78
154	"	3.96	.064	.413	.066	.130	10			4.34	.38	.535	3.67	4.08	3.26	3.42	3.62	3.83	3.73	4.46	3.82
155	"	3.91	.064	.391	.066	.130	15			4.30	.38	.520	4.18	3.86	3.24	3.40	3.59	3.78	3.69	4.35	3.78
160	"	3.96	.064	.415	.066	.130	20			4.38	.41	.544	3.87	3.56	3.26	3.43	3.63	3.83	3.74	4.02	3.82
159	"	5.58	.193	.884	.055	.248	5			5.86	.26	.930	5.01	5.00	3.87	3.97	4.47	4.83	4.78	5.04	4.82
157	"	5.58	.193	.884	.055	.248	10			5.81	.25	.666	4.81	4.97	3.87	3.60	4.20	4.34	4.64	6.11	4.51
158	"	5.58	.193	.884	.055	.248	15			5.89	.29	.938	5.25	5.64	3.87	3.98	4.47	4.84	4.78	4.55	4.84
161	"	4.00	.064	.409	.129	.193	5			4.65	.65	.620	4.47	4.63	3.28	3.54	3.72	3.94	3.79	5.07	3.92
162	"	3.94	.064	.417	.129	.193	10			4.65	.68	.620	4.34	4.20	3.25	3.54	3.70	3.94	3.77	4.64	3.92
Average		4.49		0.568		0.183				4.90		0.653	4.41	4.44	3.46	3.58	3.89	4.12	4.07	4.84	4.13
Average of 26		3.71		0.789		0.181				4.10		0.698	4.12	4.34	3.12	3.28	3.70	3.95	3.87	4.25	4.02

(a) Average of gages 2, 3 and 4
(b) " " " 1 to 5 incl.
(c) " " " 2, 3 and 5
(d) Gage 2 to gage 4
(e) " 1 " " 4
(f) " 1 " " 5
(g) Computed with reference to crest depth using V_1, V_2, d_1 & d_c

the wave velocities on tables 6 to 11, inclusive, the velocity is that of either the toe or crest, whichever could better be determined, and the one used is designated in the heading of column (12) of tables 6 to 9, corresponding to increment and decrement waves, and in column (16) for triangular waves. For rectangular waves, columns (14) and (15) of table 11 give, respectively, the velocity of the wave front and that of the wave crest. In case of waves whose longitudinal profiles were initially rectangular, the wave quickly changed to a triangular form and, as will be seen from table 11, the crest velocity, column (15), was in general greater than the velocity of the wave front, column (14); in other words, the crest of the wave, after it became triangular, traveled from an initial position near the back face of the original rectangular wave, toward the position of the front face. In case of positive increment waves formed by instantaneous increse of discharge, tables 6-a and 6-b, column (10) gives the wave depth at the cope. It will be noted that this is in general considerably less than the depth d_2 corresponding to neutral flow with the final rate of discharge. The depth at the cope of a wave is somewhat difficult to determine and more or less uncertain, especially where waves approach the breaking point. The wave velocities given in columns (15) to (18), tables 6-a and 6-b, are computed with reference to the ultimate stable depth and velocity, excepting in case of triangular waves (table 10), for which velocities computed both on the basis of stable depths (v_2) and on the basis of the crest depths (v_1) are given in the table.

With reference to triangular and rectangular waves, tables 10 and 11, the calculated velocity was obtained in each case by separately computing the velocity of the front and back faces of the wave by the given formula and taking the average. Table 10 shows the computed velocities for triangular waves for both crest depth d_c and stable depth d_2.

In case of triangular and rectangular waves, the initial and final depths and velocities were d_1v_1, and d_2v_2. The wave with stable depth and velocity d_2v_2 was superposed on the initial steady flow, since the depth and velocity, after the wave passed, returned to the original values d_1 and v_1. Combining the formulas for increment and decrement waves traveling downstream gives:

Bazin and Darcy (A):

$$u = \frac{1}{2}\left[\sqrt{gd_2} + \sqrt{gd_1} + \frac{3}{5}(v_1 + v_2)\right]. \tag{26}$$

Bazin and Darcy (B):

$$u = \frac{1}{2}\left[\sqrt{gd_2} + \sqrt{gd_1} + v_1 + v_2\right]. \tag{27}$$

Leach and King:

$$u = \frac{1}{2}\left[\sqrt{\frac{g}{2}\,\frac{d_2}{d_1}(d_2 + d_1)} + \sqrt{\frac{g}{2}\,\frac{d_1}{d_2}(d_1 + d_2)} + v_1 + v_2\right]. \tag{28}$$

Koch and Carstanjen:

$$u = \frac{1}{2}\left[\sqrt{gd_1} + \sqrt{gd_2} + v_1 + v_2\right]. \tag{29}$$

In computing velocities for crest depths, d_c is to be substituted for d_2 in these equations.

The average crest stage as observed in the course of travel of the wave through the channel is always less than the depth for steady flow, and in general the agreement of the observed velocity of the toe of the wave with the calculated velocity by the different equations is better where the calculated velocity is based on the observed crest depth. Both cases arise in practice. For example, if it is desired to determine in advance the velocity of an increment wave resulting from release of an increased volume of water to a channel for a definite time and at definite rates of increase or decrease, the depth corresponding to the crest rate of inflow can readily be determined from the rating curve of the cross-section, and this quantity can be used in computing the wave velocity. Under these conditions the ultimate stable depth only is known in advance. In predicting flood stages at points downstream, where observed crest depths are given at points upstream, neither the crest rate of flow nor the depth corresponding thereto may be known, and in this case computation of crest velocity can in general only be predicated on the observed data, namely, the actual crest depth.

FORCHEIMER'S AND SEDDON'S FORMULAS - Forcheimer has given a formula for crest velocities of waves of translation in rectangular channels which can readily be reduced to the form

$$u = \frac{1}{w} \frac{q_2 - q_1}{d_2 - d_1}. \qquad (30)$$

The values of q_2 and q_1 are those pertaining to steady flow at the actual depths d_2 and d_1. This equation relates to waves subject exclusively to friction control, since q is a function of d, dependent upon the channel characteristics.

In 1899 and agin in 1900, James A. Seddon[4] published the equation

$$u = \frac{1}{w} \frac{dq}{dh} \tag{31}$$

for the crest velocity of waves subject to friction control. Neither the Forcheimer nor the Seddon equation takes into account changes in momentum.

Experiments on triangular and rectangular waves were carried out only with the channel bottom level but rating curves for the level channel are give on figure 5. Using these, crest velocities were computed by Forcheimer's formula for triangular waves in terms both of stable depth d_2 and observed crest depth d_c, with results as shown in columns (26) and (27) of table 10, and for stable depths d_2 for rectangular waves, as shown in column (21), table 11. Since the discharge is assumed to be the same for both cases, but $d_c - d_1$ is less than $d_2 - d_1$, Forcheimer's formula gives higher velocities in terms of crest depth than in terms of stable depth for triangular waves.

Seddon's formula, if applied to the same cases, would give velocities of the same order as those given by Forcheimer's formula except for rectangular waves. For these the Seddon equation gives

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4 Seddon, James A. River hydraulics, Trans. Amer. Soc. Civil Engin., vol. 43, pp. 179-243, June 1900.

much too low velocities. The Forcheimer formula applied to triangular waves gives velocities much below the observed velocities when the computation is based on stable depths but considerably above the observed velocities when the computation is based on crest depth. For rectangular waves, using stable depth, the Forcheimer equation gives velocities of the right order and in better agreement with the observed velocities than those obtained by the other formulas used, in two groups of experiments, but gives velocities considerably below the observed velocities in the first group of experiments shown on table 11. These results may have little significance as the experiments on rectangular waves were all carried out in a level flume.

GENERAL SUMMARY - Table 12 gives a general summary containing averages of the observed data and computed velocities and ratios of the computed to observed velocities for all the experiments of each series. In general there were several groups of experiments in a given series. The results are as a rule consistent as between the individual groups and the average of all the experiments, as is shown by the following comparison for instantaneous increment waves, table 13.

TABLE 12 - CHANNEL WAVE EXPERIMENTS - GENERAL SUMMARY

Type of wave	No. of Expts.	d_1	v_1	d_2	v_2	Obserd u	$\sqrt{gd.}$	$\sqrt{gd_2}$	Bazin - Darcy $u=\sqrt{gd_2}+\frac{3}{5}v_1$: u	Bazin - Darcy $u=\sqrt{gd_2}+\frac{3}{5}v_1$: Ratio $\frac{col.10}{col.7}$	Bazin - Darcy $u=\sqrt{gd_2}+v_1$: u	Bazin - Darcy $u=\sqrt{gd_2}+v_1$: Ratio $\frac{col.12}{col.7}$	Leach - King: u	Leach - King: Ratio $\frac{col.14}{col.7}$	Koch - Carstanjen: u	Koch - Carstanjen: Ratio $\frac{col.16}{col.7}$	Ratio $\frac{col.14}{col.16}$
		Inches	f.p.s.	Inches	f.p.s.	f.p.s.			f.p.s.		f.p.s.		f.p.s.		f.p.s.		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Instantaneous increase	34	3.08	1.68	3.78	1.96	(a) 4.69	2.80	3.13	(c) 4.13	0.880	(c) 4.79	1.021	(c) 5.00	1.066	(c) 4.68	0.998	1.068
Instantaneous decrease	33	3.55	1.93	2.82	1.67	(b) 5.13	3.03	2.67	3.85	0.751	4.63	0.903	4.48	0.873	4.77	.930	0.939
Gradual increase	20	4.10	0.90	4.91	1.27	(a) 4.18	3.18	3.57	4.11	0.983	4.47	1.069	4.77	1.118	4.36	1.043	1.093
Gradual decrease	20	4.86	1.30	3.96	0.98	(b) 4.52	3.57	3.18	3.96	0.876	4.48	0.993	4.30	0.951	4.63	1.024	0.929
Triangular-Stable depth	15	4.88	0.63	5.73	0.99	(a) 4.11	3.60	3.91	4.24	1.032	4.57	1.110	4.59	1.117	4.41	1.073	1.041
-Crest depth	15	4.88	0.63	5.31	0.87	4.11	3.60	3.77	4.13	1.005	4.43	1.079	4.50	1.095	4.35	1.058	1.034
Rectangular	26	3.71	0.79	4.10	0.70	(b) 4.34	3.12	3.28	3.70	0.853	4.02	0.927	3.95	0.910	3.87	0.892	1.021

(a) Toe of wave (b) Velocity of crost (c) Computed for stable flow, using v_1, v_2, d_1, and d_2

Table 13 - Compairson of different groups of experiment with instantaneous increase in discharge.

Group	Slope of Flume	Number of experi-ments.	Ratio Leach-King / Observed	Ratio Koch-Carstanjen / Observed
(1)	(2)	(3)	(4)	(5)
Series A - 4-inch end weir	Level	2	1.110	1.045
" A - 3- " "	"	4	1.091	1.011
" A - 1-1/2" "	"	4	1.148	1.061
" D - 1-1/2" "	0.00102	2	1.338	1.227
" D - Open end	0.00102	3	1.128	1.018
" E - " "	0.00186	3	1.060	0.974
" C - " "	0.00392	5	1.100	1.022
" B- " "	0.00736	9	0.988	0.955
" B' - " "	0.00790	2	0.940	0.908
Total		34	1.066	0.998

The ratios of the computed velocities by different formulas to the observed velocities, are given on table 14. All computed velocities used in preparing this table are those for stable depths.

Table 14 - Wave velocity ratios by different formulas; averages of all experiments.

Type of wave	Bazin-Darcy (B)	Leach-King	Koch-Carstanjen	Forcheimer
(1)	(2)	(3)	(4)	(5)
	Velocity ratios, Computed/Observed			
Instantaneous increase	1.021	1.066	0.998	0.623
Instantaneous decrease	0.903	0.873	0.930	0.517
Gradual increase	1.069	1.118	1.043	0.560
Gradual decrease	0.993	0.951	1.024	0.488
Triangular--				
Stable depth	1.110	1.117	1.073	0.615
Crest "	1.079	1.095	1.058	1.230
Rectangular [1]	0.927	0.910	0.892	0.980
Average of all	1.014	1.019	1.003	0.716

[1] Computed velocities based on stable depths.

Table 14 gives the following results:

1. There is some tendency for the momentum equations used to give too high velocities for increment waves and too low velocities for decrement waves.

2. Taken altogether, there is little choice between the Leach-King, Bazin-Darcy equation (B) and Koch-Carstanjen equations, although

the accuracy of the computed wave velocities as compared with the measured velocities for different types of waves increases slightly in the order named.

3. There is some definite tendency in case of increment waves for the ratio of the observed to the computed velocity to increase as the wave length increases, i.e., proceeding from instantaneous increments through gradual increments to triangular waves, thus affording evidence of increased effect of channel friction as the wave length increases.

4. The Bazin-Darcy equation (B), Leach-King and Koch-Carstanjen equations all give higher than the observed velocities for triangular waves, computed either in terms of crest depth or stable depth. The computed velocities are more nearly correct where crest depth is used than where stable depth is used. These facts may be taken as further reflecting the effect of channel friction. It is, however, to be noted that the observed velocities used in case of triangular waves are those of the toe of the wave, which is quite certainly less than the velocity of the wave crest, to which the computed velocities apply.

The data given in table 14 are based on velocities computed from stable depths except for triangular waves, for which both stable and crest depths are used, and if actual crest depth had been used, the computed velocities would have been lower; hence it may be that the Bazin-Darcy formula (B) and the Leach-King formula would give

more nearly the correct velocities than the Koch-Carstanjen formula if computations were based on crest depths, although the Koch-Carstanjen formula gives more nearly the true velocities where computations are based on stable depth. However, in the case of triangular waves, the Koch-Carstanjen formula still gives slightly more accurate results than the other two formulas named, where the velocity is computed in terms of crest depth.

5. It is interesting to note that the Forcheimer formula, which applies to waves subject to friction control, gives, when used in terms of stable depths, velocities much velow the observed velocities except in the case of the longer waves, namely, triangular and rectangular waves. For rectangular waves the Forcheimer formula gives velocities in terms of stable depth much below the observed, and in terms of crest depth, considerably above the observed velocities. For rectangular waves the Forcheimer formula gives results higher than and more nearly correct than those given by any of the other three formulas when the computation is based on stable depths. The formula is, however, intended to apply to actual, not to stable, depths.

WAVE PROFILE - From the preceding discussion it appears that departures of calculated from observed wave velocities are due in a considerable degree to:

1. The fact that the velocities are computed for stable depths, whereas the wave form is always modified and flattened as it continues downstream.

2. The effect of channel friction.

It appears probable that channel friction plays the leading role in the transformation of the wave crest as it travels downstream.

While it is probable that more accurate values of calculated velocities of momentum waves could be obtained if crest depths were used, their use involves the difficult problem of evaluating the change of crest form in terms of the distance the wave has traveled from its point of origin. No attempt has been made to correlate the wave form and profile with the distance it has traveled.

Hydrographs of the waves as they passed all of the gages are on file at the author's laboratory, and figures 10 to 15 inclusive show sample hydrographs of several different waves of each type covered by the experiments.

IMPULSE WAVES AND INCREMENT AND DECREMENT WAVES IN STILL WATER - Theory indicates that if an impulse wave is generated by abruptly inserting a plunger in still water of depth d_1 or abruptly withdrawing it, an impulse wave will travel away from the point of generation with a velocity $\sqrt{gd_2}$. Similarly, if an outlet is abruptly opened at the end of a tank or reservoir containing still water, a decrement wave will travel upstream and an increment wave will travel downstream from the outlet, each with a velocity $\sqrt{gd_2}$.

Table 15 and figure 16 show the results of two series of experiments made at the author's laboratory in 1926 to determine the velocity of impulse waves in still water in the experimental channel. The

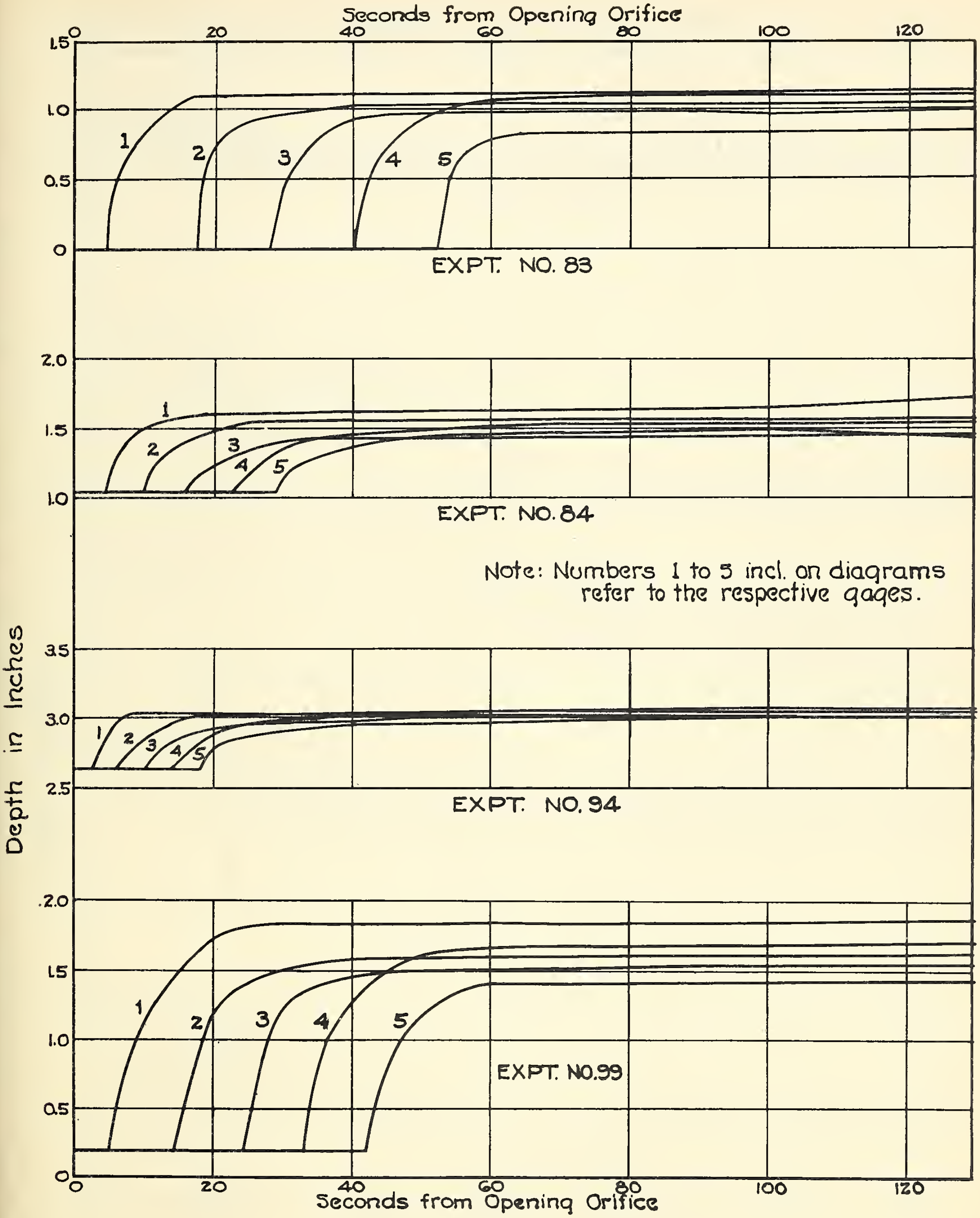


FIG. 10 ~ SAMPLES OF GRAPHS OF CHANNEL WAVE EXPERIMENTS FOR INSTANTANEOUS INCREASE

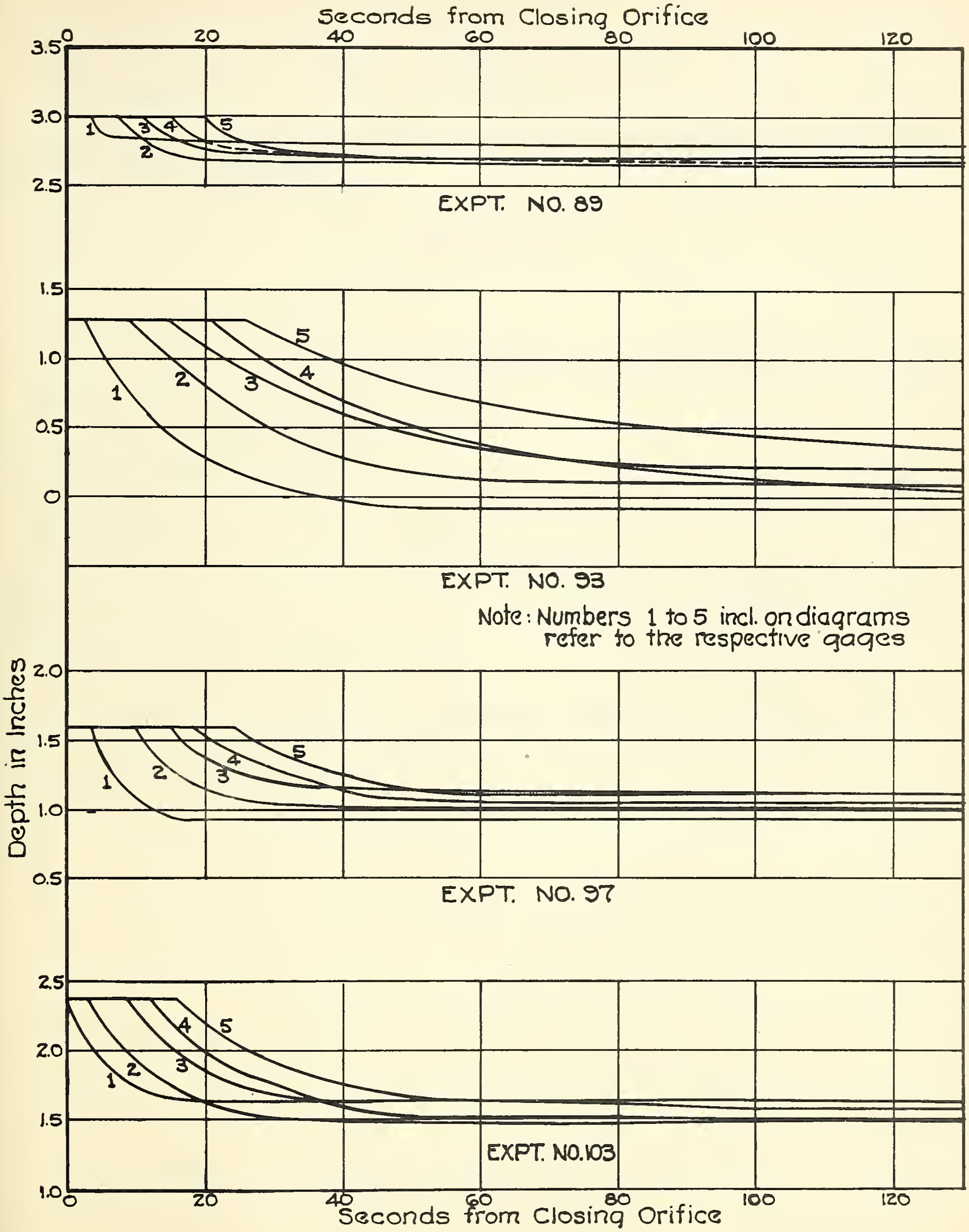


FIG. 11 ~ SAMPLES OF GRAPHS OF CHANNEL WAVE EXPERIMENTS FOR INSTANTANEOUS DECREASE

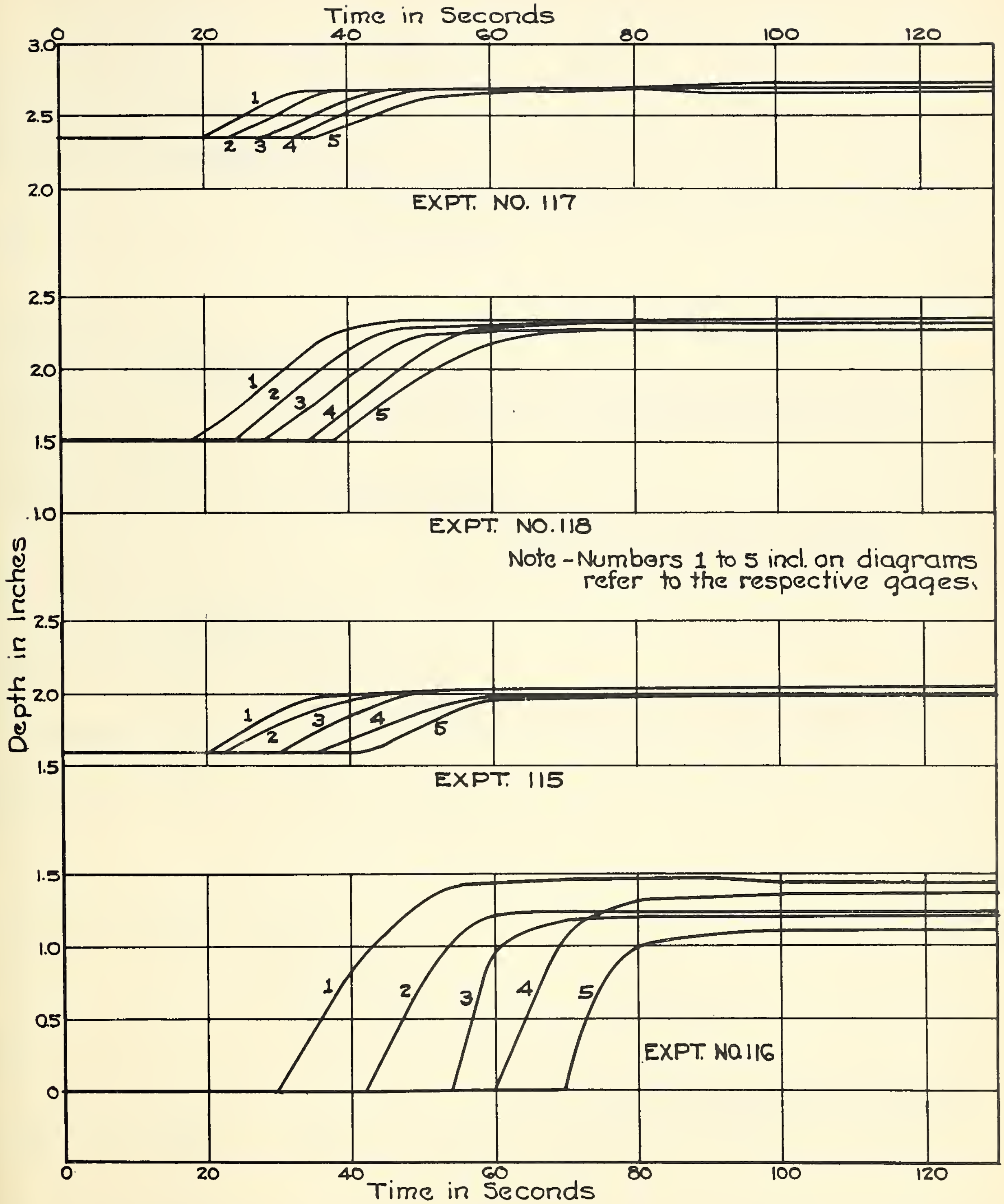


FIG. 12 – SAMPLES OF GRAPHS OF CHANNEL WAVE EXPERIMENTS FOR GRADUAL INCREASE.

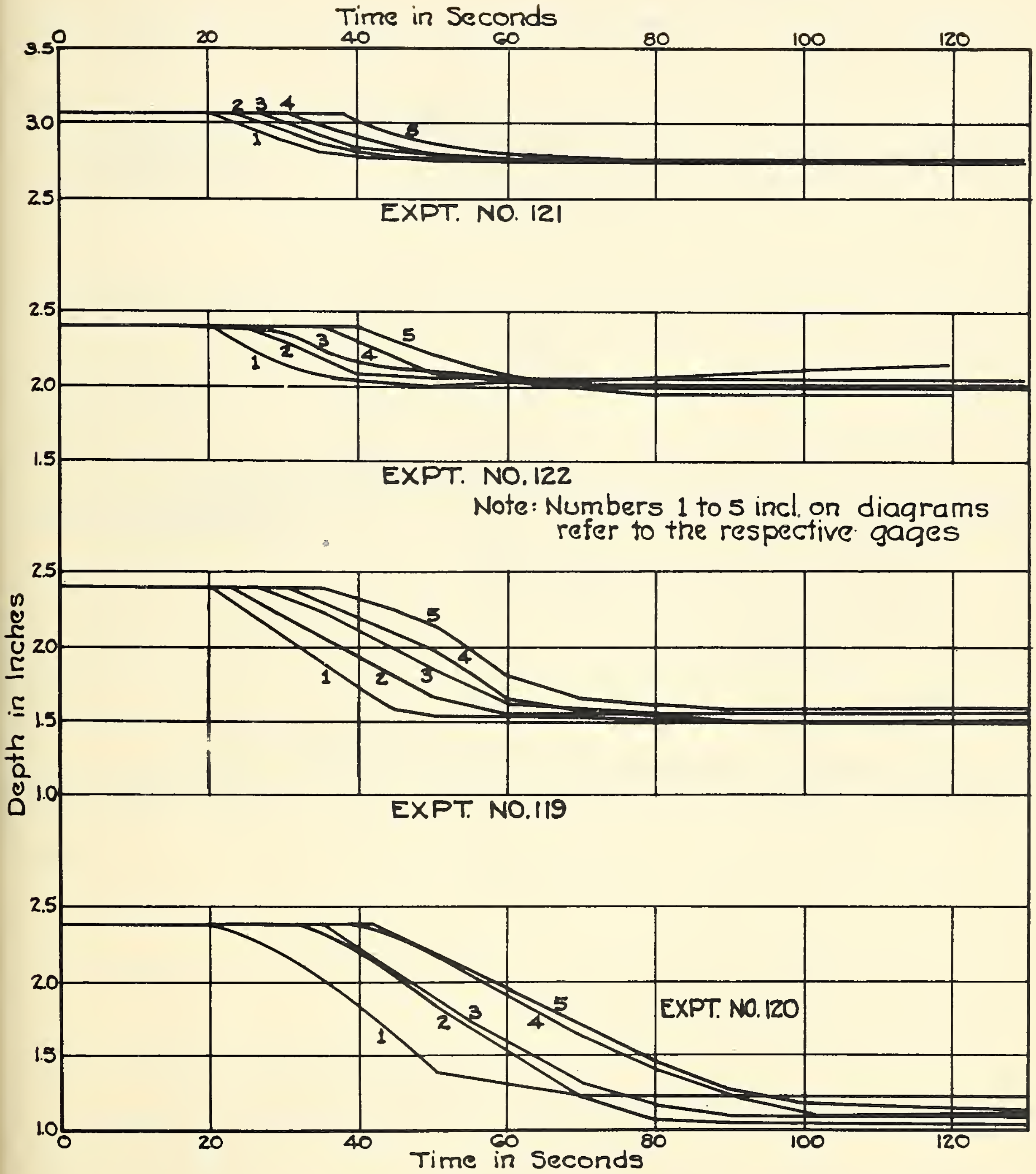


FIG. 13 ~ SAMPLES OF GRAPHS OF CHANNEL WAVE EXPERIMENTS FOR GRADUAL DECREASE

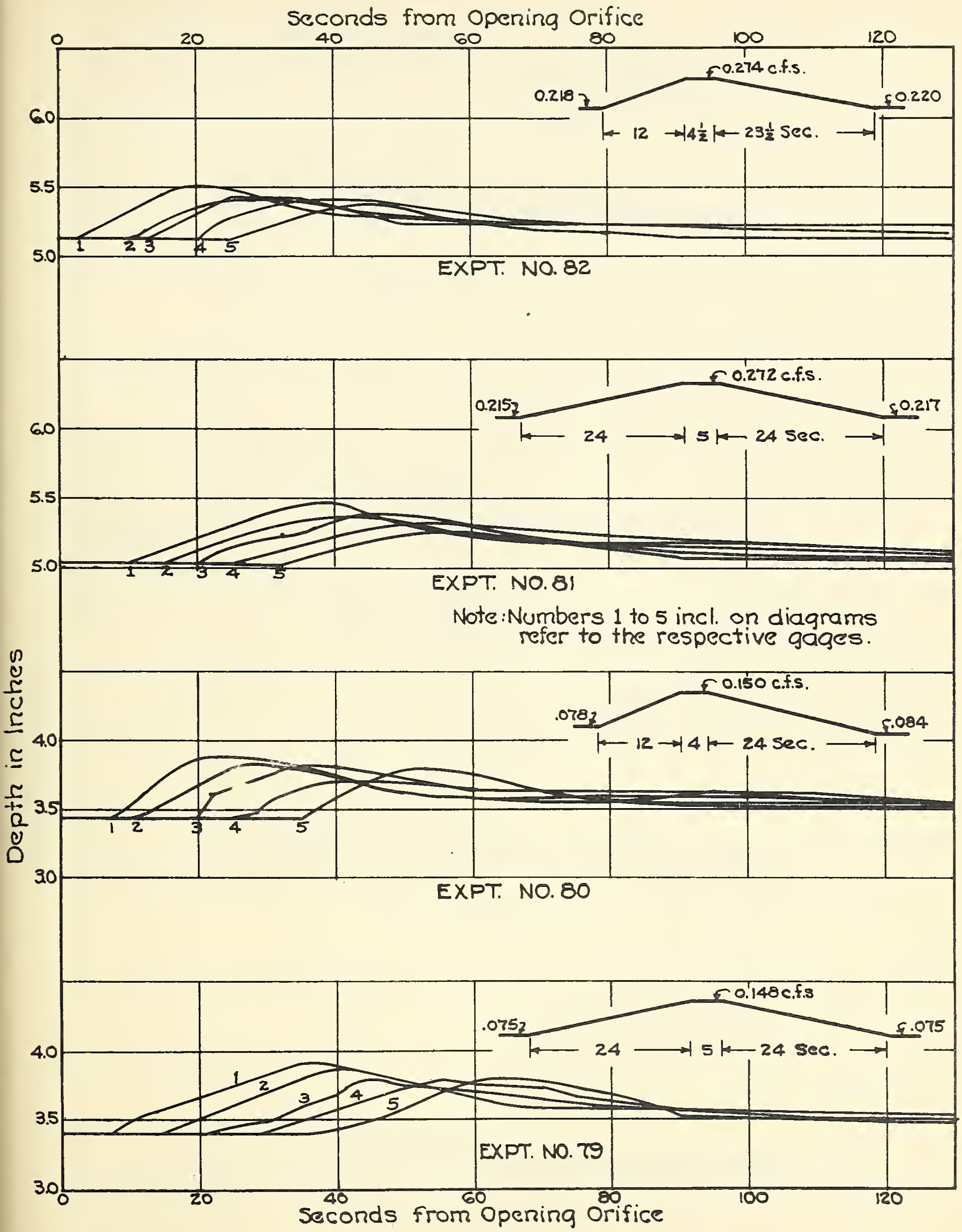


FIG. 14~SAMPLES OF GRAPHS OF CHANNEL WAVE EXPERIMENTS FOR TRIANGULAR WAVES

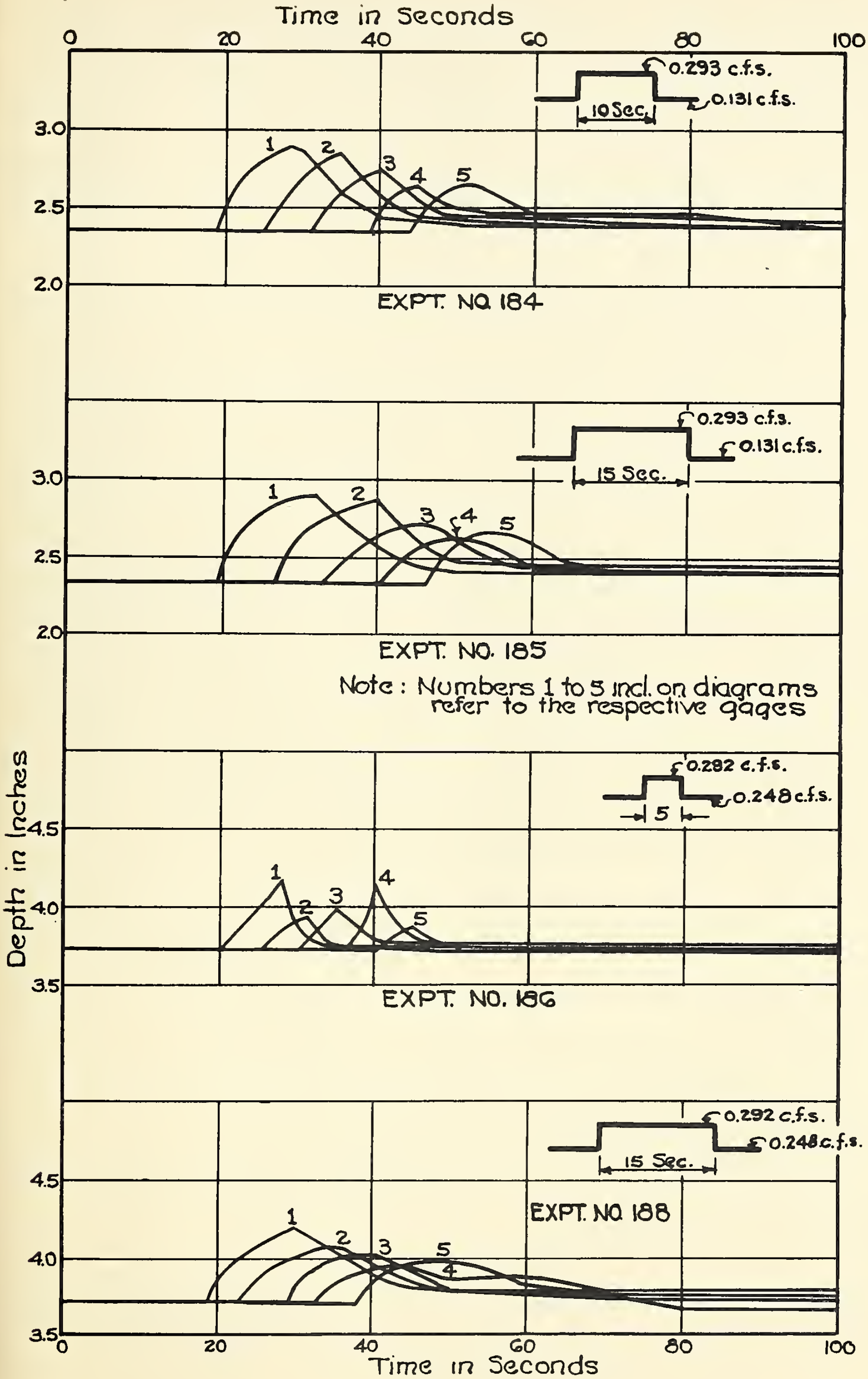
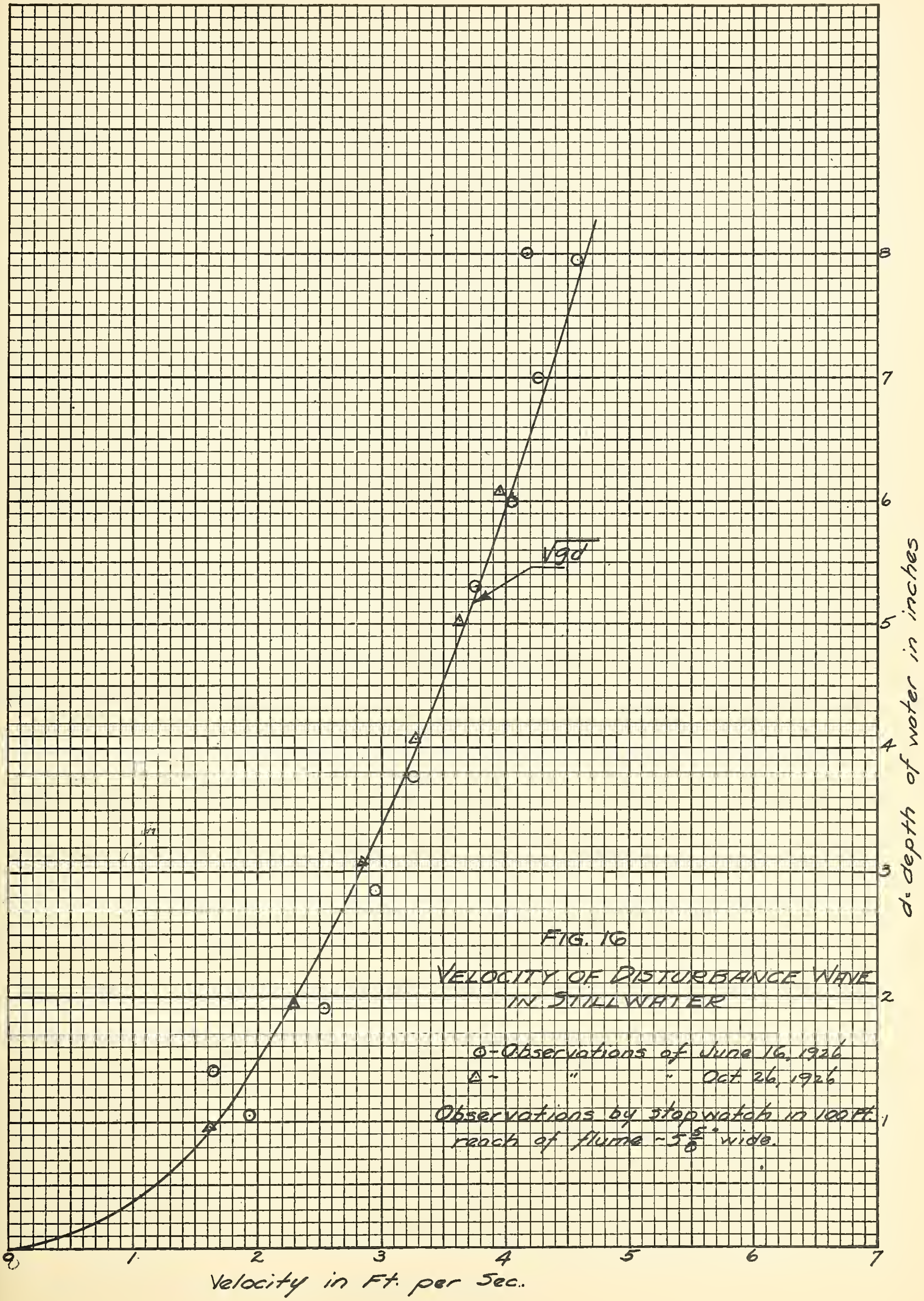


FIG. 15 ~ SAMPLES OF GRAPHS OF CHANNEL WAVE EXPERIMENTS FOR RECTANGULAR WAVES



VELOCITY OF DISTURBANCE WAVE IN STILLWATER
⊙-Observations of June 16, 1926
△- " " Oct. 26, 1926
Observations by stopwatch in 100 Ft. reach of flume ~5 5/8" wide.
$\sqrt{gd}$
d = depth of water in inches
Velocity in Ft. per Sec.
0
1
2
3
4
5
6
7
8

FIG. 16

waves were produced by dropping a plunger into the water, and the velocity is that, determined from the time, measured by a stop watch, required for the waves to travel 100 feet from gage 1 to gage 5. In this case the height of the wave relative to the initial depth d_1 was slight and was not separately measured. The velocities have been computed in terms of the initial depth d_1, as shown by column (5) of table 15. The differences between the observed and computed velocities are shown in column (6). For the first series of experiments, June 16, 1926, the means of the observed and computed velocities are identical. For the second series, October 22, 1926, the means of the observed and computed velocities are nearly identical. In general the agreement between the observed and computed velocities is extremely close for the individual experiments.

Another series of experiments was carried out, as shown by table 16, to determine the relation between the observed and computed velocities of decrement waves in still water inthe experimental channel. In this case the theoretical velocity is gd_2, as given by column (7). The difference between the observed and computed velocities is shown in column (9). While there are considerable differences between the observed and computed velocities for individual experiments, the averages for the whole series are in good agreement, the mean of observed velocities being 3.91 feet per second and the mean of the computed velocities 3.86 feet per second.

Many experiments on velocity of increment waves in still water were carried out by Bazin and Darcy in their experimental channel 2 meters in width. The solid line on figure 17 above the calculated velocities, for different values of d_2, in meters, and the plotted points show the means of the different series of experiments.

Table 15 - Channel Wave Experiments - Impulse Waves

Run No.	Depth of water in flume d inches	Time for wave to move 100 ft. secs.	Velocity u f.p.s.	$\sqrt{gd}$	Difference $\sqrt{gd}$ - u
(1)	(2)	(3)	(4)	(5)	(6)
June 16, 1926					
1	1.05	52.2	1.93	1.67	-0.26
2	1.40	61.5	1.63	1.94	+0.31
3	1.90	39.6	2.53	2.25	-0.28
4	2.82	33.8	2.96	2.75	-0.21
5	3.78	30.6	3.27	3.18	-0.09
6	5.30	26.6	3.76	3.77	+0.01
7	6.00	24.7	4.05	4.01	-0.04
8	8.00	24.0	4.17	4.63	+0.46
9	7.95	21.8	4.59	4.62	+0.03
10	7.00	23.5	4.26	4.33	+0.07
			3.32	3.32	
Oct. 22, 1926					
1	6.08	25.30	3.95	4.04	+0.09
2	5.01	27.63	3.62	3.67	+0.05
3	4.08	30.37	3.29	3.31	+0.02
4	3.09	31.86	2.87	2.88	+0.01
5	1.94	43.43	2.30	2.28	-0.02
6	0.95	62.00	1.61	1.59	-0.02
			2.94	2.96	

Wave produced by dropping plunger into water .
Travel of wave from gage 1 to gage 5, 100 ft.,
timed with stop watch.

From the experimental data described it appears that it is now well established that the velocity of either an impulse wave or an increment or decrement wave in still water is given accurately by the equation

$$u = \sqrt{gd_2}.$$

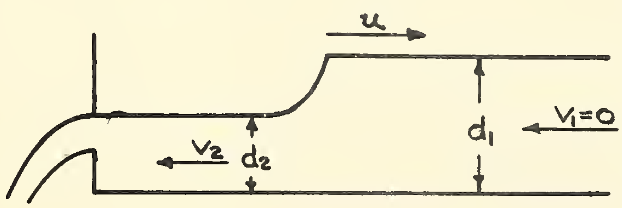
This equation, as pointed out by Bazin and Darcy, probably gives somewhat too high results for decrement waves unless of small height relative to the initial depth. Furthermore, it does not apply to and gives too large results if the wave crest breaks or forms a whitecap.

TABLE 16

CHANNEL WAVE EXPERIMENTS ~ DECREMENT WAVES

Initial velocity is zero.

Expt. No.	Opening at end of flume	Observed data: d_1 Inches	Observed data: d_2 Inches	Observed data: u f.p.s.	Drop in water surface d_1-d_2	$\sqrt{gd_2}$	Diff. $\sqrt{gd_2}-u$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
5[(a)]	3″ × 31/32″ orifice	7.41	7.00	4.67	0.41	4.34	-0.33
11	3″ × 2 1/2″ "	7.43	6.67	3.99	.76	4.23	+0.24
3[(b)]	3″ × 17/32″ "	7.19	6.97	4.55	.22	4.33	-0.22
14	4″ weir	7.22	6.42	4.32	.80	4.16	-0.16
15	4″ "	7.18	5.73	3.81	1.45	3.93	+0.12
9	3″ × 1 1/32″ orifice	6.34	5.91	4.02	.43	3.99	-0.03
12	3″ × 2 1/2″ "	6.38	5.65	3.76	.73	3.90	+0.14
13[(d)]	end open	6.07	4.37±	3.71	1.70±	3.43	-0.28
6	3″ × 31/32″ orifice	4.93	4.58	3.44	.35	3.51	+0.07
4[(c)]	3″ × 17/32″ "	4.37	4.16	3.79	.21	3.34	-0.45
10	3″ × 1 1/32″ "	4.37	4.04	3.01	.33	3.30	+0.29
	Average			3.91		3.86	



(a) Gage 5 not recording; Gage 2 off bearing.

(b) Gate jammed and time of opening uncertain within 2 seconds.

(c) Leaf across orifice after opening.

(d) Gage 5 submerged owing to high velocity at end of flume.

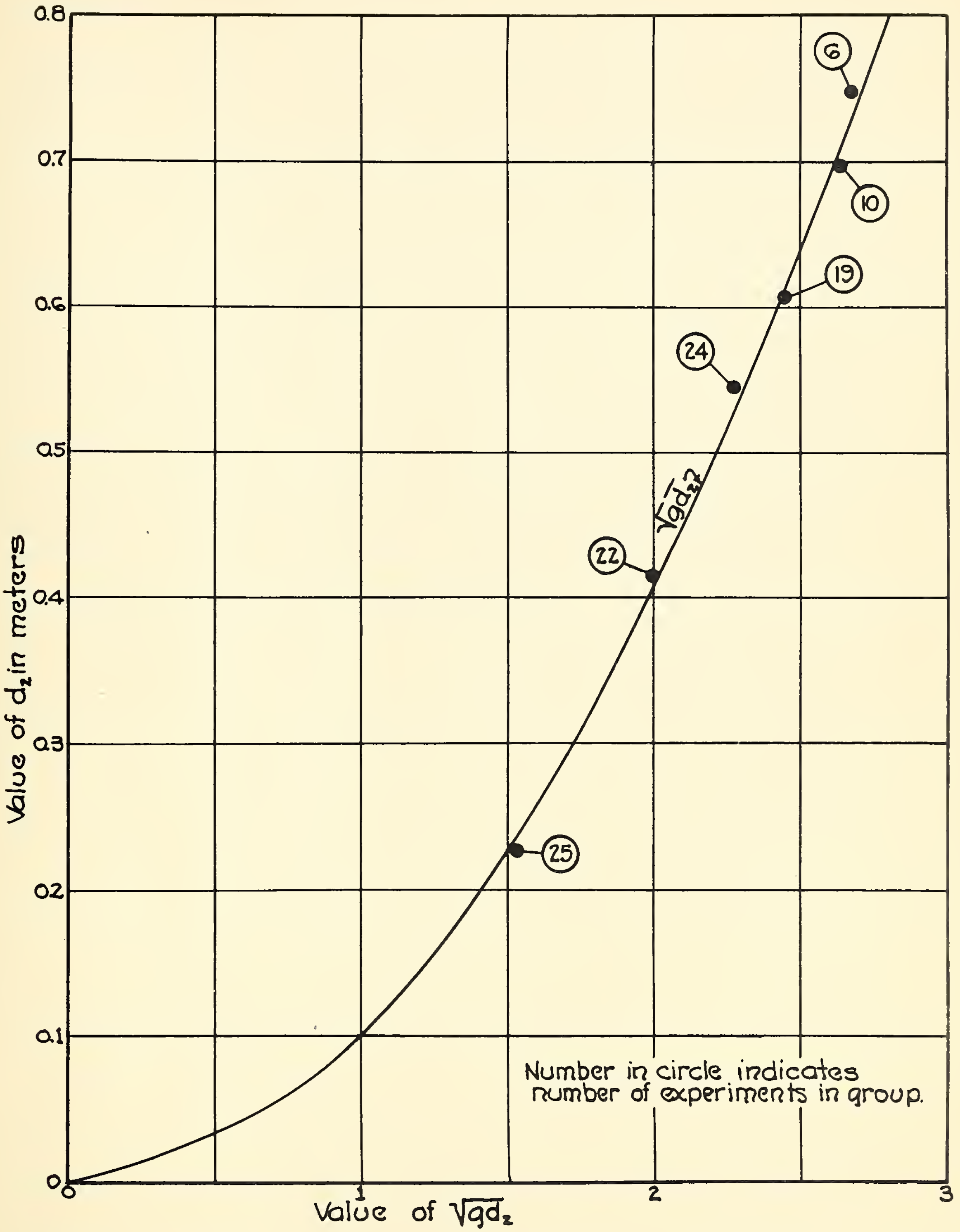


FIG. 17 ~ BAZIN–DARCY EXPERIMENTS. WAVES PROPAGATED IN STILL WATER. SERIES 1 ~ EXPTS. 1–11, INCL.

ALTERNATING WAVES - If a channel or reservoir has an outlet, and the outlet opening is more or less abruptly increased or decreased, a series of alternating waves is set up which travel back and forth throughout the length of the reservoir. These waves may be definite and regular in case of a channel of uniform width and depth but are less definite and regular in case of a reservoir of varying width and depth, and especially if there is continuous inflow. Important cases of the occurrence of alternating waves arise for reservoirs at power dams when the flow through the turbines is suddenly increased or decreased. A similar phenomenon occurs in case of a partial or complete failure of a dam impounding water.

The characteristics of the series of alternating waves may be determined as follows. For purposes of illustration the case of abrupt opening of the outlet gates of a reservoir without inflow will be considered. Similar reasoning applies to the case of abrupt closing or decrease of the opening of the outlets of a reservoir. Increasing the outlet, however, produces decrement waves upstream from the dam traveling initially upstream. Decreasing the outlet produces increment waves which travel initially upstream.

Consider the case where an outlet is abruptly opened in a reservoir without inflow. The water level just upstream from the dam suddenly drops by an amount Δd, yet to be determined. Then a decrement wave of height Δd travels upstream, with a velocity

$$u = \sqrt{g\ (d_1 - \Delta d)} = \sqrt{gd_2}.$$

A uniform, level channel is assumed, of width w and initial water depth d_1, and the outflow is assumed to take place through an orifice having its center at a depth h below the water surface, so that $h_2 = d_2 \pm b$, where b is a constant. The outflow rate through the orifice is then

$$q = CA\sqrt{2g\,(h_2 + b)} \qquad (32)$$

but as the decrement wave travels upstream it removes a volume of water per second, also equal to q, and expressed by the equation

$$q = wu\,\Delta d. \qquad (33)$$

It is obvious that the value of Δd will be such that the value of q determined by the two equations will be the same. Substituting the value of $u = \sqrt{g\,(d_1 - \Delta d)}$ gives

$$w\,.\,\Delta d\,.\,\sqrt{gd_2} = CA\sqrt{2gd_2 \pm b.} \qquad (34)$$

A similar equation can of course be written out where the outflow takes place over a weir or through a pipe, by expressing the discharge rate, as given by the right hand member, in terms of the initial head. As above given for outflow through an orifice, this is a cubic equation, in which the only unknown quantity is d_2, the value of which can be obtained by ordinary methods of solution of such equations. A solution is in general most readily obtained by computing values of both members of the equation for assumed values of d_2, plotting the results in terms of d_2 and finding the required value of d_2 at the point of intersection of the curves.

As the decrement wave travels upstream it virtually strips off from the surface of the reservoir a layer of water of thickness Δd, the head and discharge at the outlet remaining meanwhile constant. When a decrement wave has reached the head of the reservoir it returns to the outlet, the outflow rate and head at the outlet remaining the same as before. The value of Δd is, however, less for the returning wave because of the decreased value of d_2. Calling $\Delta' d$ the value of this quantity for the return wave, this value can readily be obtained by the same method as before. As a decrement wave returns to the outlet it strips off a layer of water of thickness $\Delta' d$. When it reaches the outlet or dam, a second decrement wave starts upstream but in this case the head and consequently the discharge at the outlet are decreased, and the value of Δd applying to this case, and which may be called $\Delta'' d$, is again changed. In this way successive layers of water are stripped off from the surface, and changes of head and discharge at the outlet take place abruptly each time the wave completes a circuit - in other words, the discharge takes place by definite quanta instead of varying or decreasing uniformly with time, as it is assumed to do in most ordinary calculations of outflow from reservoirs.

The thicknesses of the layers stripped from the reservoir surface successively as the wave traverses from outlet to head or back, are not uniform but vary in a somewhat complex manner with the conditions, for the reason that the thickness of each layer, or Δd, depends on the velocity of wave travel, and this decreases as the initial depth

decreases. The velocity decreases and this tends to increase the value of Δd as the depth decreases. The value of Δd also depends on the outflow rate, decreasing with the latter as successive layers are stripped from the reservoir surface. These opposite effects tend, as shown by the hypothetical example given below, to produce a minimum thickness of the layer Δd at some depth of draw-down.

The process is illustrated by a hypothetical case of emptying a canal 100 feet long, with level bottom, width w, 1 foot, initial depth d_1, 1 foot, with an outlet at its lower end having an area of 0.05 square foot, coefficient C = 0.60, the center of the outlet being 0.8 foot below the initial water surface. The thicknesses of successive layers Δd and the computed velocities of the wave as it travels alternately up and downstream are shown by figure 18.

Thus far in the discussion of the subject the effect of channel friction has been entirely neglected. An experiment was carried out, the results of which are shown by figure 19, with an initial depth of slightly over 6 inches in the experimental channel.

The lines on figure 19 show the resulting hydrographs of the alternating waves as they passed successively the different gages, which were 25 feet apart. When the outlet was opened, the decrement wave first reached gage 5, 15 feet upstream from the outlet. An abrupt drop of about one-half inch occurred at this gage, then the level remained nearly constant to the point a'. The time aa' represents that required for the wave to travel upstream 115 feet to the head of the

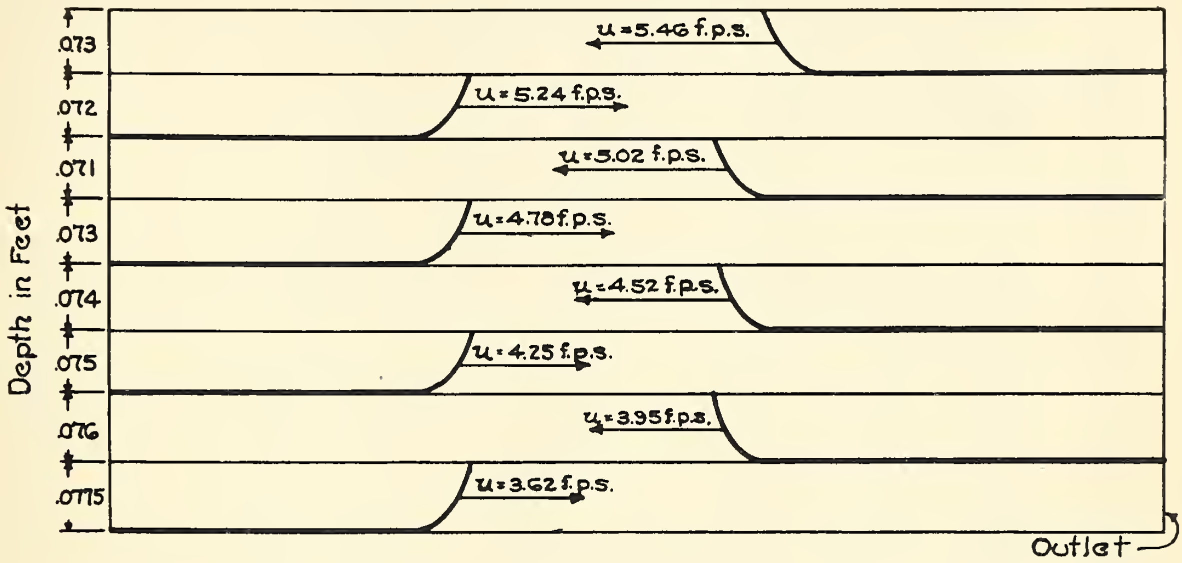


FIG. 18 - ALTERNATING WAVES

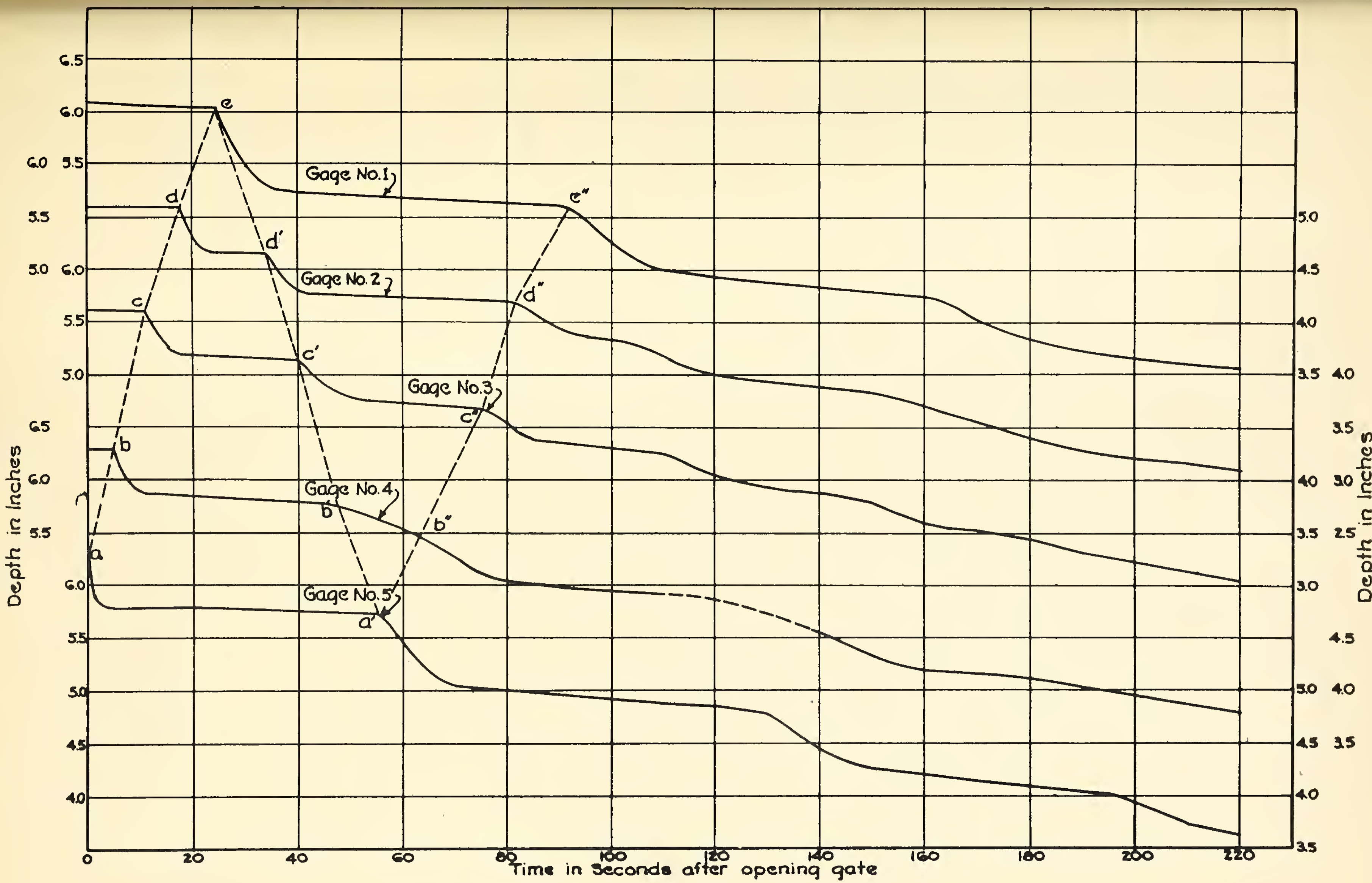


FIG. 19.-CHANNEL WAVE EXPERIMENTS. GRAPHS OF WAVE, INSTANTANEOUS DRAWDOWN IN STILL WATER
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channel and return the same distance to gage 5. Gage 4 was 25 feet upstream from gage 5, and the time bb' is that required for the water to travel a total distance of 180 feet to the head of the canal and back. Similarly, b'b" represents the time elapsed after the first return wave had reached gage 4 until the second decrement wave had traveled upstream to gage 4, a total distance of 80 feet. cc" represents the time required for the return wave and the second decrement wave to travel from gage 3 to the outlet and back, a total distance of 130 feet, and so on.

Referring to figure 19, it will be noted that after the first two cycles, the wave became less definite in form and the water level did not remain constant between successive passages of a given decrement of flow but, instead, the water surface in the canal tended to assume a uniform slope and the character of the outflow changed gradually from that of flow by quanta to ordinary hydraulic flow, with a sloping surface or drop-down curve in the canal.

WAVE ADVANCING IN INITIALLY EMPTY CHANNEL - In the operation of reservoirs, water is often released into an initially empty channel and the question of the time required for it to arrive at a power plant, irrigation head-works or other point of use is often important. Several experiments were made by admitting water to the initially empty channel, with different slopes, as shown on table 17. In these experiments, the velocity of the toe only was determined.

TABLE 17.– VELOCITY OF WAVE ~ INITIALLY EMPTY CHANNEL

Expt. No	Slope of Flume S_c	Velocity of Toe u_t f.p.s.	Observed Crest Depth d_c Inches	Compt'd Crest Depth $= \frac{144 Q_2}{5\frac{5}{8} u_t}$	Stable Depth d_2 Inches	Ratio $\frac{\text{col. 5}}{\text{col. 6}}$	Velocity V_2 f.p.s.	$\sqrt{gd_2}$	$V_2+\sqrt{gd_2}$	$\frac{3}{5}V_2+\sqrt{gd_2}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
169	.00102	1.338	(a)	1.245	1.44	0.865	1.13	1.97	3.10	2.65
170	"	1.845	(a)	1.860	2.26	.823	1.40	2.46	3.86	3.30
179	"	1.960	(a)	1.736	2.30	.754	1.41	2.49	3.90	3.34
171	"	2.257	(a)	2.190	3.03	.723	1.55	2.86	4.41	3.79
123	.00392	1.678	1.320	1.250	1.37	.963 (b)	1.52	1.92	3.44	2.83
83	.00736	2.125	1.02	.890	1.02	1.000 (b)	1.86	1.65	3.51	2.77
99	"	2.674	1.44	1.320	1.55	.930 (b)	2.35	2.04	4.39	3.45
116	.00790	2.562	1.538	1.375	1.57	.980 (b)	2.25	2.05	4.30	3.40
105	.00736	3.253	2.020	1.660	2.00	1.010 (b)	2.72	2.32	5.04	3.95
109	"	3.870	2.952	2.595	2.98	.990 (b)	3.37	2.83	6.20	4.85
Average		2.36						2.26	4.21	3.43

(a) Time of wave only measured.

(b) col. 4/col. 6

For the flatter slopes the observed velocities are less, and for the steeper slopes greater, than $\sqrt{gd_2}$. This indicates that the stable velocities affect the velocity of the wave front or toe. The wave velocities are, however, less than either $v_2 \sqrt{gd_2}$ or $v_2 + \frac{3}{5}\sqrt{gd_2}$ as given by the Bazin-Darcy formula. Evidently the velocity of the toe of a wave traveling down an initially empty channel is some function both of $\sqrt{gd_2}$ and v_2 and is not, as has commonly been supposed, limited to a value never exceeding $\sqrt{gd_2}$.

Jeffreys[5] has concluded that wave trains in shallow channels may travel with a maximum limiting velocity of $2\sqrt{gd_2}$.

The physical explanation of Jeffreys' analytical results seems to be that the maximum velocity for uniform flow in a channel is $\sqrt{gd_2}$ but that a wave train may ride superposed on such flow and with a velocity $\sqrt{gd_2}$ relative thereto, so that the actual velocity of the wave train relative to the earth would be, as a maximum limit,

$$u = 2\sqrt{gd_2}.$$

It is probably true that an unsupported solitary wave in an otherwise empty channel could not travel with a velocity exceeding $\sqrt{gd_2}$, but in the case to which the author's experiments apply, namely, the introduction of a constant rate of flow into an initially dry channel, the resulting wave front or bore is not unsupported. Apparently the bore can travel of itself at a velocity $\sqrt{gd_2}$ and, in addition, it may be pushed along by the water which is continually accumulating

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[5] Jeffreys, Harold. The flow of water in an inclined channel of rectangular section, Phil. Mag., ser. 6, vol. 49, pp. 793-803.

behind it, so that, under conditions where channel friction overbalances the pushing effect of the water behind the bore, the velocity of the bore front in a dry channel may be somewhat less than $\sqrt{gd_2}$, whereas if the slope is sufficiently steep and the conditions such that the pushing effect of the water behind the bore exceeds the frictional resistance at the wave front, the velocity of the wave front may exceed $\sqrt{gd_2}$. At least, this physical interpretation affords a consistent explanation of the results of the author's experiments and points to lines of further experiment, which it is hoped may lead to a definite solution of this important problem.

INSTANTANEOUS SHUT-DOWN - The converse of the preceding case occurs when the flow into a channel is instantly shut down. The question then arises, how long before the first effect of the shut-down will be felt at a given point downstream. Experiments with instantaneous complete shut-down are given in table 18. In this case it seems obvious that the first effect will travel downstream with a velocity equal to the sum of the initial velocity plus $\sqrt{gd_1}$, and the experiments confirm this conclusion.

Table 18 - Instantaneous complete shut-down

Experi-ment Number	Slope of Flume	Initial Condition			Final depth	u	$\sqrt{gd_1}$	$\sqrt{gd_1}+v_1$
		d_1	Q_1	v_1				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Series C - Open end							
134	0.00392	1.20	0.070	1.49	0	3.14[1] 3.58	1.79	3.28
	Series B - Open end							
93	0.00736	1.28	0.108	2.16	0	4.09	1.86	4.02
104	"	1.58	0.140	2.27	0	4.83	2.06	4.33
106	"	2.05	0.211	2.63	0	5.15	2.34	4.97
110	"	3.02	0.390	3.30	0	6.08	2.85	6.15

[1] Actual u uncertain, within these limits.

RELATION OF WAVE VELOCITIES FOR WAVES SUBJECT, RESPECTIVELY, TO MOMENTUM CONTROL AND CHANNEL FRICTION CONTROL - One purpose of these experiments has been to discover lines of further research on channel waves which would be most likely to lead to useful results. It has been shown that the Koch and Carstanjen equation when applied to the author's experiments gives results generally within a few percent of the observed velocities. Taking the averages of all the experiments on a given type of wave, the difference between the observed and calculated velocities is positive for some groups, negative for

others. Considering positive waves only table 14 shows the following relation of observed velocity of the toe of the wave to velocity calculated by Koch and Carstanjon's formula:

Instantaneous increment	1.002
Gradual "	0.959
Triangular waves-	
Stable depth	0.932
Crest "	0.945
Rectangular waves	1.121

The observed and calculated velocities are nearly identical for instantaneous increment waves, and this is the case for which the formula used is derived and to which it should be most strictly applicable.

The final depths used in calculating velocities are the stable depths in all cases except the computation labeled "Triangular waves - Crest depth." For gradual increments and triangular waves the lower relative velocities strongly imply the effect of channel friction increasing as the wave length increases.

Somewhat similar results follow from a comparison of observed velocities with those calculated by the Bazin and Darcy formula, although less consistent than by the Koch and Carstanjon formula.

The anomalous result for rectangular waves, i.e., observed higher than computed velocity, holds true whether the velocity ratio is computed for observed crest velocity or observed velocity of wave front (toe), and also occurs for all the formulas. It also holds in spite

of the fact that the calculated velocities are based on stable depths, d_2, whereas initially rectangular waves were quickly transformed into triangular waves in the experiment channel.

The experiments on rectangular waves, while apparently consistent among themselves, were performed in a level channel where variations of depth due to back-water and drop-down curves occurred. This condition and the rapid transformation of wave form may have affected the validity of the results. While rectangular waves, although quickly transformed into triangular waves, have more abrupt wave fronts than waves initially triangular in profile, still the wave front is not more abrupt than that of instantaneous increment waves. The fact that rectangular waves showed relatively higher velocities than any other form covered by the experiments leaves the validity of this series of experiments somewhat in doubt.

Presumably channel friction effects increase with wave length. In the author's experiments, with gradual increase or decrease of discharge, and with triangular and rectangular waves, the wave length did not in general exceed 1,000 to 2,000 times the wave height. In case of natural flood waves in stream channels, the entire wave length is seldom in the stream channel at one time. The true wave length, i.e., the distance from the toe of the wave to the heel at the time when the normal flow is restored at the head of the drainage basin, as it would be if the wave was confined in a continuous channel, would then be 1-1/2 to 2 or more times the total channel length.

Assuming, for purpose of illustration, a wave length equal to the channel length, then even for a 20-foot rise in a channel 100 miles long, the ratio of wave length to wave height would be $\frac{100 \text{ miles}}{20 \text{ feet}}$ or about 25,000.

The fact that Forcheimer's formula gives nearly the correct velocity for rectangular waves is somewhat surprising but may be merely accidental. Forcheimer's formula, if applicable at all to waves such as those covered by the experiments, should apply best to triangular waves with velocities computed in terms of crest depths. For this type of wave the ratio of the observed velocity to that given by Forcheimer's formula is 0.814, and if computed in terms of stable depths, the ratio of the observed velocity to that given by Forcheimer's formula is 1.62. Since these experimental waves apparently were not of sufficient length to be subject wholly to friction control, close agreement between the observed velocities and those given by Forcheimer's formula is not to be expected.

These results point directly to the need of additional experimental data on a uniform channel of sufficient length so that experiments can be carried out on waves having lengths relative to their height corresponding to those of natural flood waves in river channels. The practical difficulty of such experiments arises obviously from the great length of channel required. However, there appears no other way to evaluate the laws of flood movement in the wide range of cases intermediate between that of waves subject directly to momentum control and that of waves subject exclusively to channel friction control.

CONCLUSIONS - These experiments point to the following conclusions:

1. Increment waves in the experimental channel were little affected by channel friction and their velocities are quite accurately given by the momentum equations, particularly the Koch-Carstanjen equation, and a little less accurately by the Bazin-Darcy and Leach-King equations.

2. For a gradual rise or for triangular waves, the observed velocities fall below the calculated velocities based on stable depth. The difference increases as the ratio of wave length to wave height increases, indicating an increasing effect of channel friction with increasing wave length.

3. It seems probable that the effect of channel friction varies as a function of the ratio of wave length to wave height and also as some function of the ratio of the hydraulic radius to the wave height.

4. The existing momentum formulas do not give as good results applied to decrement waves as when applied to increment waves. This difference is probably related to the fact, pointed out by Bazin and Darcy, that decrement waves never occur singly but are followed by trains of secondary waves. Little evidence of this effect was noted in the author's experiments.

5. There is a wide range of conditions from those of pure momentum waves to those of long waves subject exclusively or chiefly to friction control. Existing formulas apply well to the former. The only available formula applicable to waves subject to friction control is that of Forcheimer.[6] While Forcheimer's formula apparently applies well

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[6] Forcheimer, Philipp. Water movement in translatory waves (Ger.), Zeit. für Gewässerkunde, vol, 6, part 6, pp. 321-339, 1904.

to the author's experiments on rectangular waves, if the computation is based on stable depths, it is much less satisfactory in case of triangular waves, to which, apparently, it should apply best, and much further experimental work is needed since it is evident that there is no sharp line of demarcation between momentum waves and waves subject to friction control. For intermediate cases the wave velocity may partake of both and lie between the values given by the formulas for the two cases. Natural river flood waves are approximately triangular waves but usually with a much greater ratio of wave length to wave height than in the case of the triangular waves covered by the author's experiments.

6. The author's experiments and those of Bazin and Darcy show that both impulse and increment and decrement waves in still water have the same velocities, and that the velocity is given accurately by the momentum equation, $u = \sqrt{gd_2}$.

7. The experiments show that an instantaneous increment wave travels down an empty channel with a velocity which is not precisely equal to $\sqrt{gd_2}$ but is less for channels with flat, and greater with channels with steep, slopes.

8. The experiments also show that when the inflow to a channel is abruptly shut off, the velocity with which the first effect is transmitted downstream is given accurately by the equation $u = v_1 + \sqrt{gd_1}$, within the limits of the cases covered by the experiments.

9. An experiment is given which clearly illustrates the characteristics of alternating waves in case of emptying a reservoir and it is shown:

(a) That the layers of water stripped successively off from the reservoir surface are not uniform but there is a layer of minimum thickness at some particular depth of draw-down.

(b) If the water is of sufficient depth, the outflow rate remains constant while a given wave is traveling from the outlet to the head of the reservoir and back, and changes by definite quanta for each complete wave circuit. For shallower depths the regimen of outflow by alternating waves gradually breaks down, approaching a condition of a sloping water surface in the reservoir and a uniformly decreasing outflow rate, as in the case of ordinary hydraulic flow.

10. While these are probably the most comprehensive experiments thus far performed, only one size and length of channel was used, and perhaps the most useful service rendered by these experiments is to call attention to the need of additional experiments covering:

(a) All the types of waves here considered.

(b) Experiments with several channesl of increasing cross-section, so that the ratio of hydraulic radius of channel to wave height may be used as a controlled independent variable in evaluating the effect of channel friction.

(c) Experiments on waves having a range of ratio of wave length to wave height, covering all cases from instantaneous increment waves

to waves having ratios of length to height of the same order as in natural river channels.

(d) It seems advisable in future experiments to start with a channel of small cross-section but of much greater length than that used in the author's experiments. This is apparently necessary in order to obtain, within practicable limits, waves subject mainly to friction control. Larger channel sections and shorter channels may then be used progressively in covering the transition from friction waves to momentum waves.